THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

VOLUME LXV MARCH 1927 NUMBER 2

A STUDY OF SOLAR MOTION BY HARMONIC ANALYSIS

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ABSTRACT

Determination of solar motion.—A method of obtaining the solar apex and velocity by graphical-harmonic analysis is here presented, which, it is claimed, excels Airy's least-squares method in its simplicity, ease, and adaptability to available data. The theory of the analysis is discussed and its further possibilities in the study of stellar parallaxes and group motion are pointed out.

To demonstrate the method, the radial motion of 476 stars was obtained from the publication of the *Dominion Astrophysical Observatory*, **2**, No. 1, and their corresponding proper motion, from Boss's *Preliminary General Catalogue*. From these, the solar apex was determined with the following results:

$$\alpha = 272^\circ, \quad \delta = 33^\circ.7, \quad V = 24.8 \text{ km/sec.}, \quad \pi_{av} = 0.0092.$$

To show the relative accuracy of the harmonic analysis and the least-squares analysis, the radial velocities of sixty stars, of declination about 60°, were used; by harmonic analysis the apex and velocity were found to be

$$\alpha = 266^\circ, \qquad \delta = 41^\circ.5, \qquad V = 20.5 \text{ km/sec.};$$

by least squares,

$$\alpha = 256^\circ, \qquad \delta = 60^\circ, \qquad V = 17.3 \text{ km/sec.},$$

the former being nearer to the values obtained from the most recent and extensive studies of the determination of the solar apex.

Concerning the ether-drift experiment.—Inasmuch as the foregoing method is the outcome of the investigation of the recent data on the ether-drift experiment (*Science*, April 30, 1926), a discussion of the latter is given.

INTRODUCTION

Nearly all of the recent determinations of the probable position of the apex and the speed of the solar motion have employed Airy's least-squares method.

It has been pointed out by a number of writers that if, in the

determination of the position of the solar apex, the stars to be used are grouped in small areas, the majority of the stars in one area play a small part in the result, since the presence of a few large proper motions virtually determines the mean proper motion of the area.[1]

The nature of the process of the least-squares solution does not enable the investigator to know readily and easily what effect a certain star or a group of stars will produce in the result. It might be added that the method involves considerable labor, which limits the extent of the investigation.

In this paper we propose a method free from the objectionable features of Airy's least-squares solution; it is amply accurate, considering the precision of the data available, and is much less laborious; it provides a simple method for the study of the group motion of stars and of stellar parallaxes.

THEORETICAL CONSIDERATIONS

It is assumed here that the peculiar motions of stars are at random.

Denote by α_a and δ_a the right ascension and declination of the solar motion, and by V its speed (in kilometers per second). Then the angular velocity of the solar motion, V'' (seconds of arc per year), will be equal to $V\pi_a/4.74$, when π_a is the average parallax of the stars used in the determination. Also denote the proper motion in right ascension and declination of a star whose right ascension is α and declination δ by μ_α and μ_δ (measured in seconds of arc per year), its radial velocity by ρ (kilometers per second) and its parallax by π (seconds of arc).

Assuming the directions of ρ, μ_α, and μ_δ through the star as the axes of a system of reference, we have from trigonometry:

$$-\rho = V\cos\delta_a\cos\delta\cos(\alpha_a-\alpha)+V\sin\delta_a\sin\delta\,, \tag{1}$$

$$-\mu_\alpha\cos\delta = V''\cos\delta_a\sin(\alpha_a-\alpha)\,, \tag{2}$$

$$-\mu_\delta = -V''\cos\delta_a\sin\delta\cos(\alpha_a-\alpha)+V''\sin\delta_a\cos\delta\,. \tag{3}$$

Let δ, the declination of the star, be constant; that is, consider the motions of a band of stars of the same declination but having

[1] A. S. Eddington, *Stellar Movements and the Structure of the Universe*, p. 80.

any value of right ascension, α. Then, since V, V'', δ, δ_a, and α_a are constants, it is seen that $\mu_\alpha \cos \delta$, μ_δ, and ρ are simple harmonic functions of α.

If values of ρ are plotted against α, that is, if the radial velocity of each star is plotted as ordinate and its right ascension as abscissa, then an approximate sine curve will be formed of the type $\rho = P_\rho + A_\rho \sin(\alpha + \phi_\rho)$, where P_ρ is the "average ordinate" or the displacement of the axis of the curve from the zero line, A_ρ is the amplitude of the curve, and ϕ_ρ its phase. These three quantities are represented by:

$$P_\rho = -V \sin \delta_a \sin \delta \,, \tag{4}$$

$$A_\rho = V \cos \delta_a \cos \delta \,, \tag{5}$$

$$\phi_\rho = 270^\circ - \alpha_a \,. \tag{6}$$

Similarly, if the curves for $\mu_\alpha \cos \delta$ and μ_δ are drawn, their average ordinates, amplitudes, and phases are:

$$P_{\mu_\alpha} = 0 \,, \tag{7}$$

$$A_{\mu_\alpha} = V'' \cos \delta_a \,, \tag{8}$$

$$\phi_{\mu_\alpha} = -\alpha_a \tag{9}$$

$$P_{\mu_\delta} = -V'' \sin \delta_a \cos \delta \,, \tag{10}$$

$$A_{\mu_\delta} = V'' \cos \delta_a \sin \delta \,, \tag{11}$$

$$\phi_{\mu_\delta} = 90^\circ - \alpha_a \,. \tag{12}$$

If, instead of δ, α is made constant; that is, if, instead of using a declination band of stars, a meridian band is used, then from equations (1) and (3) it can be seen that ρ and μ_δ are simple harmonic functions of δ. If curves are plotted, the average ordinates, amplitudes, and phases are, in this case:

$$P'_\rho = 0 \,, \tag{13}$$

$$A'_\rho = V\sqrt{\sin^2 \delta_a + \cos^2 \delta_a \cos^2 (\alpha - \alpha_a)} \,, \tag{14}$$

$$\phi'_\rho = \tan^{-1} [\cot \delta_a \cos (\alpha - \alpha_a)] \,, \tag{15}$$

$$P'_{\mu_\delta} = 0 \tag{16}$$

$$A'_{\mu_\delta} = V''\sqrt{\sin^2 \delta_a + \cos^2 \delta_a \cos^2 (\alpha - \alpha_a)} \,, \tag{17}$$

$$\phi'_{\mu_\delta} = \tan^{-1}[\tan \delta_a \sec (\alpha - \alpha_a)] \,. \tag{18}$$

The corresponding meridian band from equation (2) will be a straight line displaced from the axis by:

$$P'_{\mu_\alpha} = V'' \cos \delta_a \sin (\alpha - \alpha_a) . \qquad (19)$$

METHOD OF ANALYSIS

In the actual analysis three curves are drawn with right ascension as abscissa and with ρ, $\mu_\alpha \cos \delta$, and μ_δ, respectively, as ordinates. The stars are grouped in declination bands 10° wide. Corresponding curves for meridian bands one hour wide are drawn. In practice the scale used was 400 mm for twenty-four hours of right ascension, 2 mm for 1 km per second radial velocity, 2 mm for 0".0015 proper motion in right ascension, and 1.5 mm for 0".001 proper motion in declination.

The average ordinate can be found by obtaining the area in square millimeters, with a planimeter, between the curve and the zero line, from zero to twenty-four hours, and dividing the area by 400 mm, the length of the base.

The phase and amplitude are obtained by means of the Henrici harmonic analyzer,[1] which is a device for finding the coefficients of the Fourier series:

$$y = A_1 \sin (\theta + \phi_1) + A_2 \sin (2\theta + \phi_2) + A_3 \sin (3\theta + \phi_3) + \ldots\ldots$$

It is necessary to use only the integrators for the first component, since the theoretical curve is a simple sine curve.[2]

ADVANTAGES OF METHOD

It is possible to plot and analyze the three curves (equations [1], [2], and [3]) for a declination band of one hundred stars in one afternoon. The proper and radial motions are at all times visible on the curves, and discrepancies can be noted and errors corrected quickly and easily. If a separate determination for a group of stars is desired, this can readily be accomplished by joining the points corresponding to these stars by straight lines and analyzing the

[1] O. Henrici, "On a New Harmonic Analysis," *Philosophical Magazine*, Vol. **38**, 1894.

[2] D. C. Miller, "The Henrici Harmonic Analyzer," *Journal of the Franklin Institute*, September, 1916.

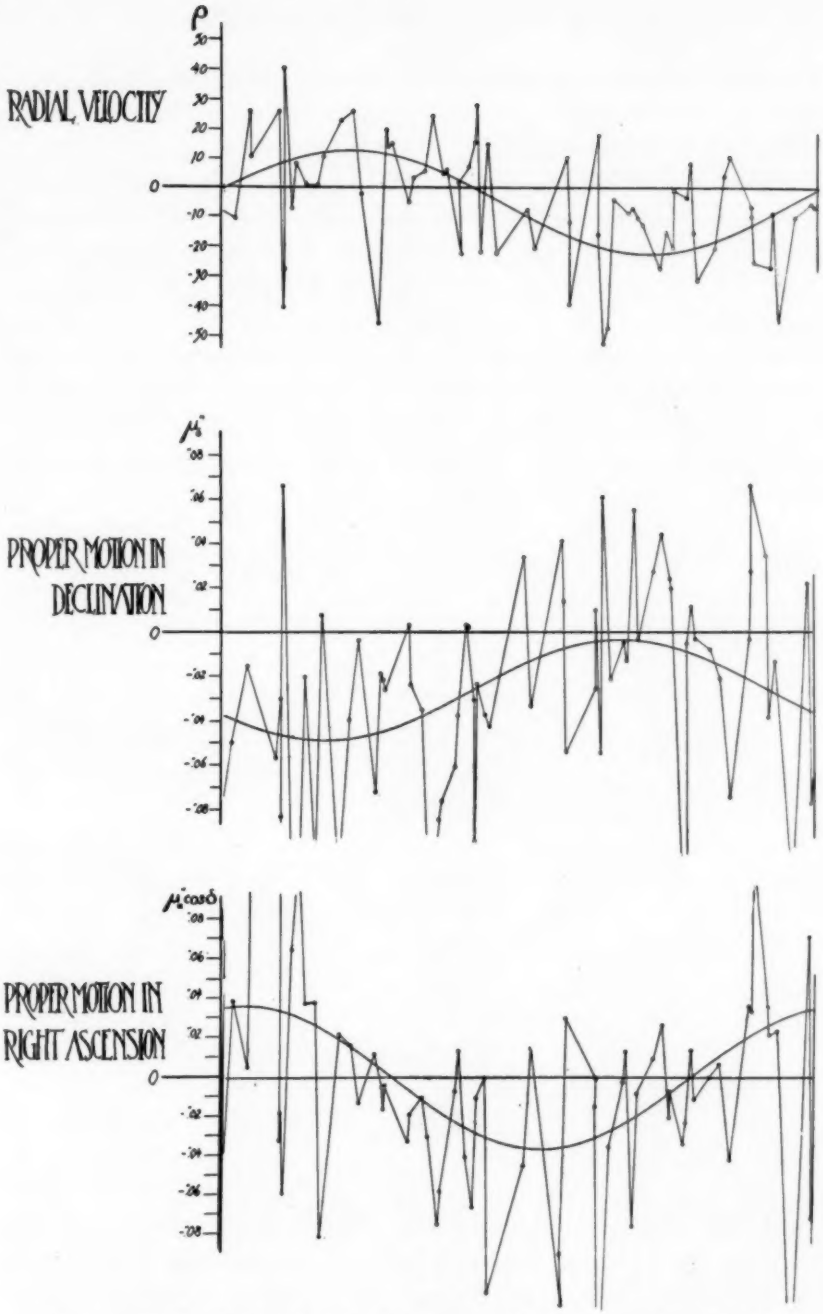


FIG. 1.—Declination band, 25°–35°. The smooth sine curves have the phase, amplitude, and average ordinate obtained by harmonic analysis of the irregular data curves.

resulting curve, a matter of a few minutes' time. Also, if it is desired to note the effect of omitting any star in the determination, it is evident that it can easily be accomplished.

Large discrepancies, which are always present in the data available and which greatly affect the final result when the least-squares solution is employed, have very little effect when the harmonic analyzer is used, especially when the stars are close together in right ascension.

If a correction in the form of an additive constant, or of a sine or cosine function of α, is to be applied to the data, as is the case with stars from Boss's *Catalogue*, such a correction can be applied to the results of the analyzed curve, without the need of correcting each individual proper motion or radial velocity.

Some irregularities in the weights of individual observations are likely to occur if the spacing in right ascension is irregular, but a judicious averaging of points can be made which will obviate this to some extent.

OTHER APPLICATIONS OF THE METHOD

The determination of the solar apex is but one of the results which can be obtained from these curves by this method of analysis. Other applications are here suggested.

Draw over each data curve the sine curve representing the determined solar motion for that band. The vertical distance between this sine curve and the points representing the motion of an individual star denote by Δ_ρ, Δ_{μ_α}, and Δ_{μ_δ}; Δ_ρ indicating radial motion, Δ_{μ_α} and Δ_{μ_δ} indicating proper motion in right ascension and declination, respectively, and being considered *plus* when above the sine curve and *minus* when below. Thus we are able to get a set of three curves for each declination band from which a general idea of group motion can easily be obtained. Stars comparatively near together, whose Δ_ρ are nearly equal and of the same sign, and whose Δ_{μ_α} and Δ_{μ_δ} are of the same sign, may be considered as traveling in a group.

If the parallax of one star in a group is known, then the ratio between the Δ_μ of this star and the Δ_μ of another star of unknown parallax in the same group will be equal to the ratio of their paral-

laxes. This is, of course, possible only when observational errors are small.

If the $\mu_\alpha \cos\delta$ and μ_δ curves are analyzed, the solar velocity will be found in seconds of arc, as V''. In order to reduce this to kilometers per second, the average parallax of the stars must be known. To obtain this, advantage is taken of the properties of the ρ and μ_δ curves of any meridian band. The amplitudes of each, from equations (14) and (17), are in the ratio of $V : V''$.

The curves, in their simplicity of form and ease of analysis, show many further possibilities which cannot be developed here, but which will no doubt be of considerable help in the study of stellar motions.

SOLAR MOTION DERIVED FROM 476 STARS

Material used.—The material consists of those stars in the *Publications of the Dominion Astrophysical Observatory*[1] whose radial velocities do not exceed 50 km/sec. after being corrected for the solar motion, and whose proper motions do not exceed 0".5 per year; in all, 476 stars were used. The proper motions of the stars were obtained from Boss's *Preliminary General Catalogue* and were corrected as described on page xxviii of this *Catalogue*. Further corrections were applied to the proper motions in declination as recently proposed by Raymond.[2]

Method of computation.—The stars were arranged in declination bands lying between 0° and 5°, 5° and 15°, 15° and 25°, 25° and 35°, 35° and 45°, 45° and 55°, and 55° and 65°, giving seven bands whose average declinations were $2\frac{1}{2}°$, 10°, 20°, 30°, 40°, 50°, and 60°.

As the accuracy of the right ascension determined by this method depends on the amplitude of the curve, the amplitude of the curve was used as weight for each individual determination.

Since the radial velocities contain a constant error K_ρ which would displace the curve vertically, and which, therefore, would appear as a constant error in the P_ρ's, the results of the ρ curves were solved to obtain K_ρ, and the values were then corrected.

[1] "The Radial Velocities of 594 Stars," *op. cit.*, **2**, No. 1

[2] H. Raymond, "The Correction to the Declination System of Boss's *Preliminary General Catalogue*," *Astronomical Journal*, **36**, 17–18, 1925.

In order properly to weight the determination of δ_a, equations of the following form were used:

$$V \sin \delta_a = \frac{\Sigma P_\rho}{\Sigma \sin \delta},$$

$$V \cos \delta_a = \frac{\Sigma A_\rho}{\Sigma \cos \delta}.$$

TABLE I

DATA: DECLINATION BANDS

Declination Bands	A_ρ	P_ρ	P_ρ Corrected	ϕ_ρ	a_ρ
0°	20.0	− 0.1	1.5	345°	285°
10	22.3	− 0.5	1.1	19	251
20	21.5	+ 1.4	3.0	344	286
30	15.8	4.4	6.0	10	260
40	18.7	7.1	8.7	351	279
50	15.3	8.0	9.6	360	270
60	7.7	+11.8	13.4	4	266

TABLE II

DATA: DECLINATION BANDS

Declination Bands	A_{μ_a}	ϕ_{μ_a}	a_{μ_a}	A_{μ_δ}	P_{μ_δ}	ϕ_{μ_δ}	a_{μ_δ}
0°	0.0487	81°	279°	0.0036	0.0284	182°	268°
10	0.0409	60	300	0.0061	0.0236	196	254
20	0.0332	69	291	0.0108	0.0268	168	282
30	0.0349	81	279	0.0230	0.0258	209	241
40	0.0501	90	270	0.0331	0.0315	171	279
50	0.0266	74	286	0.0306	0.0106	194	256
60	0.0342	72	288	0.0180	0.0097	186	264

Results.—Using the material and the method of harmonic analysis as described, the results for the apex and velocity of the solar motion are as follows:

From radial motions alone:

$$a_a = 270°45',$$

$$\delta_a = 31°45',$$

$$V = 25.1 \text{ km/sec.}$$

From proper motions alone:

$$\alpha_a = 273^\circ 00' ,$$
$$\delta_a = 35^\circ 40' ,$$
$$V'' = 0''.0473 .$$

From radial motions and proper motions combined:

$$\alpha_a = 272^\circ 00' ,$$
$$\delta_a = 33^\circ 40' ,$$
$$V = 24.8 \text{ km/sec.} ,$$
$$\pi_{av} = 0''.0092 .$$

Two sets of meridian curves were drawn, one for 0° and 180°, and the other for 90° and 270°. Each band was 15° wide. The analysis gave $A'_\rho / A'_{\mu_\delta} = V/V'' = 580$, that is, $\pi_{av} = 0''.0082$.

To show the relative accuracy of the harmonic analysis and the least-squares analysis, the ρ curve for the 60° declination band, consisting of sixty stars, was used, giving by the method of harmonic analysis:

$$\alpha_a = 266^\circ , \qquad \delta_a = 41^\circ.5 , \qquad V = 20.5 \text{ km/sec.} ;$$

while the least-squares method applied to the same set of stars gives:

$$\alpha_a = 256^\circ , \qquad \delta_a = 60^\circ , \qquad V = 17.3 \text{ km/sec.}$$

The most recent and extensive determination of the solar apex, made by Dr. Ralph E. Wilson,[1] gives:

$$\alpha_a = 270^\circ.8 , \qquad \delta_a = 27^\circ.1 , \qquad V = 19.0 \text{ km/sec.} ,$$

from which it is seen that the treatment of a group of observations by harmonic analysis gives a result nearer to the values obtained from this study, the most complete which has been made.

[1] "The Solar Motion Problem," *ibid.*

SOME THEORETICAL CONSIDERATIONS OF THE SOLAR MOTION AS DETERMINED BY ETHER-DRIFT

The foregoing investigation is the outcome of the theoretical consideration of the Michelson and Morley ether-drift experiment, as it has been undertaken recently by Professor Dayton C. Miller.[1] The formulae used in the determination of the solar motion relative to the ether, from observations made by the interferometer, can be derived as follows:

The interferometer measures the projection of the apparent relative motion of the earth and of the ether, giving the azimuth A and magnitude v of this projection as functions of the sidereal time. If α and δ represent the right ascension and declination of the relative motion of the earth and the ether, and V its velocity, we have the familiar equations:

$$\cos z = \sin\delta \sin\phi + \cos\delta \cos\phi \cos t\,, \tag{20}$$

$$\sin z \cos A = +\sin\delta \cos\phi - \cos\delta \sin\phi \cos t\,, \tag{21}$$

$$\sin z \sin A = \cos\delta \sin t\,, \tag{22}$$

$$v = V \sin z\,, \tag{23}$$

$$t = \theta - \alpha\,, \tag{24}$$

where ϕ is the latitude of the observatory, z is the zenith distance of the apex, and θ is the sidereal time of the observation.

As the observed azimuth and magnitude of the drift are quite independent of each other, each gives a separate determination of the apex.

Solar apex derived from magnitude of ether-drift.—If V and its direction are assumed to be constant throughout a short period of time (about ten days), and ϕ and δ are positive, equations (20) and (23) show that v is a minimum when $t = 0$. Therefore we may write:

$$\left.\begin{aligned} \alpha &= \theta \text{ when } v \text{ is minimum,} \\ z_{min.} &= |\,\phi - \delta\,| \text{ when } v \text{ is minimum}\,, \\ v_{min.} &= V \sin|\,\phi - \delta\,|\,. \end{aligned}\right\} \tag{25}$$

[1] "Significance of the Ether-Drift Experiment of 1925 at Mount Wilson," *Science*, April 30, 1926.

If $\delta \leqq 90^\circ - \phi$, the maximum value of v occurs when $z = 90^\circ$, and therefore

$$v_{max.} = V \tag{26}$$

so that

$$\frac{v_{min.}}{v_{max.}} = \sin \mid \phi - \delta \mid$$

or

$$\delta = \phi \pm \sin^{-1}\left(\frac{v_{min.}}{v_{max.}}\right) . \tag{27}$$

If $\delta > 90^\circ - \phi$, a rough value of δ may be obtained from equation (27), or the maximum value of v occurs when $z = 180^\circ - (\phi + \delta)$, from which we obtain:

$$\frac{v_{min.}}{v_{max.}} = \frac{\sin \mid \phi - \delta \mid}{\sin (\phi + \delta)} . \tag{28}$$

From the foregoing we observe that if v is plotted against sidereal time, equations (23), (25), (26), (27), and (28) will determine the direction and velocity of the solar system relative to the ether.

The theoretical curve for v is

$$v = V \sin [\cos^{-1} (\sin \delta \sin \phi + \cos \delta \cos \phi \cos t)] . \tag{29}$$

Applying equations (25) and (28) to the observed magnitude of ether-drift (average for April, August, and September, 1925, and February, 1926) as observed by Professor Miller, the direction of the apex was found to be:

$$a = 252^\circ , \qquad \delta = 71^\circ ,$$

Solar apex derived from azimuth of ether-drift.—The azimuth A is measured from the north point, *plus* toward the east and *minus* toward the west. Since the interferometer observations cannot distinguish between azimuths 180° apart, we shall consider A numerically less than 90°.

CASE I, $\delta \geqq \phi$

It is clear that when $t=0$, v is minimum and $A=0°$. As t increases, A becomes negative, and when t becomes equal to twelve hours, A becomes zero again. For greater values of t, the azimuth becomes positive. Hence when the azimuth passes from east to west t becomes equal to 0, or $\theta=\alpha$, which corresponds to the place where the curve of azimuth against time crosses the time-axis, when the azimuth passes from a maximum to a minimum.

Dividing equation (21) by equation (22) we obtain

$$A=\cot^{-1}[-\sin\phi\cot t+\tan\delta\cos\phi\csc t]\,. \qquad (30)$$

The maximum value of A is such that

$$\sin A_{max}=\cos\delta\sec\phi\,.$$

This gives the required value of δ.

The average observed azimuth for the four epochs (April, August, and September, 1925, and February, 1926) has for an X-axis (time-axis) a line 62° below the X-axis of equation (30).

It was found necessary, therefore, to shift the axis of the observed azimuth 62° (*Science*, April 30, 1926, p. 10).

With this modification and from the foregoing discussion, the direction of the apex becomes:

$$\alpha=255°\,, \qquad \delta=70°\,.$$

The curves for magnitude and azimuth (equations [29] and [30]), while they are not simple sine curves, may be considered as harmonic for not more than three components of the Fourier series. These components were found, and the curves were synthesized.[1]

Thus α, δ, and V were computed.

CASE II, $\delta<\phi$

Here again, when $t=0$, $A=0°$. As t increases, A receives the following successive values:

$$0\,,\ldots,90°\,,-90°\,,\ldots,0°\,,\ldots,90°\,,-90°\,,\ldots,0°\,.$$

Hence when $A=0°$, $\alpha=\theta$ or $\alpha=\theta+12^h$.

[1] D. C. Miller, "A 32-Element Harmonic Synthesizer," *Journal of the Franklin Institute* (January, 1916), pp. 51–81.

The value of t when $A = 90°$ being known, equation (30), will give δ.

It is also possible to compute the north-south and east-west components of the magnitude, and thus obtain two sets of curves which correspond respectively to the μ_α and μ_δ curves (equations [2] and [3]) of the first section of this paper. Their analysis can be made in much the same manner as that of the proper-motion curves.

The writers are under obligation to Professor Dayton C. Miller for his kindness in making available the apparatus for the harmonic analysis and his inspiration and help in carrying on the work.

CASE SCHOOL OF APPLIED SCIENCE
July 1926

THE ELECTRIC-FURNACE SPECTRA OF YTTRIUM, ZIRCONIUM, AND LANTHANUM[1]

By ARTHUR S. KING and EDNA CARTER

ABSTRACT

Furnace spectra of yttrium, λ 2808–λ 7882; zirconium, λ 2792–λ 8305; lanthanum, λ 2799–λ 8347.—Temperature stages from 2000° to 2800° C. were employed to observe the *initial appearance of lines* and their *rate of increase in intensity*, supplemented by low-temperature absorption spectra in the ultra-violet. A *direct comparison with arc and spark spectra* showed the changes undergone in these sources and served to select the *enhanced lines*. The numbers of lines classified are 451 of yttrium, 858 of zirconium, and 695 of lanthanum. Many enhanced lines of yttrium and lanthanum, and some of those of zirconium, are emitted by the furnace, their intensities in furnace and arc indicating the relative energy-levels of the ionized atom. For zirconium a temperature of about 2400° was required before any considerable number of lines appeared, and for this element a large proportion of lines emitted by the arc are enhanced lines. In the lanthanum spectrum, note is made of the large number of close doublets, and some triplets, all of which have been previously listed as single lines; also of the large proportion of lines relatively stronger in the furnace than in the arc.

From an examination of the furnace, arc, and spark spectra of yttrium, zirconium, and lanthanum in the interval λ 2800 to λ 8400, data have been obtained for a classification of their lines according to the temperature required for initial appearance and the rate of change of line-intensity as the temperature is increased. Not only the lines of the neutral atom, but also, to a considerable extent, those of the ionized atom were treated in this way. The enhanced lines have been selected by a direct comparison of arc and spark spectrograms. For yttrium and lanthanum especially, sufficient ionization is present at the higher furnace temperatures to produce the more sensitive enhanced lines. The degree in which these are radiated by the furnace and by the stronger ionization of the arc should aid in placing the relative energy-levels of ionized lines just as the temperature classification of other elements has served to indicate energy-levels for the neutral atom.

EXPERIMENTAL METHOD

Furnace.—The vacuum furnace previously described[2] was used, the material being vaporized in graphite tubes of the usual dimen-

[1] *Contributions from the Mount Wilson Observatory*, No. 326.

[2] *Mt. Wilson Contr.*, No. 247; *Astrophysical Journal*, **56**, 318, 1922.

sions: 12.5 mm bore, 19 mm outside diameter, and 200 mm between the supporting clamps.

Spectrograph.—From λ 2800 to λ 5000, the second order of the 15-foot concave-grating spectrograph was employed, the scale of the plates being 1.86 A per millimeter. Beyond λ 5000, the first order of the grating was used.

Temperatures.—It was found that for yttrium and lanthanum, a temperature of 1900° C. was required to produce a fair number of lines, and at this temperature the spectrum was faint. About 2000° was therefore regularly used for the low-temperature stage, while 2200°–2300° and 2600°–2800° served for the medium and high temperatures, respectively. Zirconium is more difficult to vaporize, and because of the high temperatures required, only two stages were used —2400° for the low and 2700° for the high temperature.

Emission and absorption spectra.—As in the case of spectra previously studied, it was found that low-temperature lines in the ultraviolet appear at shorter wave-lengths in absorption than in emission, on account of the rapid weakening of the furnace emission in this region. To obtain an absorption spectrum, white light from a tungsten lamp provided with a quartz window was passed through the furnace tube while this was heated at the low-temperature stage. The lines which the low-temperature vapor absorbs were thus found, and were used up to the wave-length at which emission spectrograms for the same temperature begin to show lines. By means of the overlapping region, the intensities of the absorption lines were harmonized as well as possible with the emission lines; the emission lines alone were used as the estimation was carried toward longer waves.

Chemicals.—Salts of yttrium and lanthanum often contain impurities, especially rare earths, which give many foreign lines, with resulting disturbance by blends. An extensive set of early yttrium spectrograms was rejected on this account. We were assisted in obtaining better yttrium by Dr. W. F. Meggers, who furnished some very pure oxalate prepared by Mr. J. F. T. Berliner, of the Bureau of Chemistry, Department of Agriculture. We are indebted also to Professor B. S. Hopkins, who gave us lanthanum oxide, purified with great care in the chemical laboratory of the University of Illinois.

TABLE I

Temperature Classification of Yttrium Lines

λ (I.A.)	Arc Int.	Furnace Intensities			Class	λ (I.A.)	Arc Int.	Furnace Intensities			Class
		High Temp.	Medium Temp.	Low Temp.				High Temp.	Medium Temp.	Low Temp	
2807.77	1	3	1	.....	III A	3280.94	10	.....	.....	.....	V E
2813.61	8	5	3	tr	III	3287.24	4	2	1	.....	III
2818.84	3	2	.....	.....	IV	3327.88	150	20	12	.....	III E
2822.55	10	3	tr	.....	IV	3330.87	3	.....	.....	.....	V E
2826.29	1	2	tr	.....	IV A	3335.17	8n	.....	.....	.....	V
2835.78	1	1	.....	.....	IV	3340.39	7	3	2	.....	III
2854.42	3	.....	.....	.....	V E	3344.54	4n	.....	.....	.....	V
2856.30	1	.....	.....	.....	V	3354.58	10	{10 ? 5 ?	5 ? 2 ?	.....	III III
2886.49	15	8	3	.....	III						
2890.39	3	1	.....	.....	IV	3358.96	5	2	1	.....	III
2897.69	1	.....	.....	.....	V	3362.00	20	.....	.....	.....	V E
2898.85	tr	.....	.....	.....	V	3364.79	7	5	3	.....	III
2901.46	6	5	2	.....	III	3377.67	6	2	1 ?	.....	III
2919.06	20	10	8	4	II	3382.82	5	.....	.....	.....	V
2948.39	30	15r	12	6	II	3383.03	7	.....	.....	.....	V
2964.96	30	15r	12	6	II	3388.58	15	4	2	.....	III
2974.58	35	20r	15	9	II	3389.87	5	1 ?	.....	.....	IV ?
2980.54	2	.....	.....	.....	V E	3394.97	4	3	1	.....	III
2984.26	50	30r	20	12	II	3397.05	15	4	2	.....	III
2995.25	10	10	10	2	III	3412.46	12	5	3	.....	III
2996.93	20	12	12	3	III	3414.5	8n	4	3	.....	III
3005.24	12	10	12	2	III	3424.16	7	15	15	1 ?	III A
3018.93*	6 ?	8	3	.....	III	3431.01	6	.....	.....	.....	V
3021.70	15	20	15	8	II	3431.67	5	6	6	.....	III
3022.27	12	18	15	6	II	3433.79	6n	2	.....	.....	IV
3038.47	4	10	7	tr	III A	3437.98	8n	3	tr	.....	IV
3044.83	5	5	3	.....	III	3448.85	35	1 ?	.....	.....	V E
3045.36	20	20	20	12	II	3450.96	12	12	8	tr	III
3047.13	3	4	2	.....	III	3461.02*	4	?	?	.....	V ?
3047.38	3	3	2	.....	III	3467.88	20	1	.....	.....	V E
3056.33*	2 ?	3 ?	1 ?	—?	III ?	3471.77*	10n	3 ?	2 ?	.....	V ?
3059.52	4	5	2	.....	III	3473.18	5	5	3	.....	III
3072.37	5	5	2	.....	III	3484.05	10	15	15	2	III
3076.50	5	5	2	.....	III	3485.74	40	12	8	tr	III
3078.58	2	.....	.....	.....	V E	3496.07	150	10	7	.....	III E
3086.86	7	tr	.....	.....	V	3498.90	10n	3	1	.....	III
3091.70	15	10	8	tr?	III	3500.61	7	4	2	.....	III
3095.88	25	1	.....	.....	V E	3501.97	2	1	.....	.....	IV
3096.61	3	4	2	.....	III	3506.47d	6	3	2	.....	III
3103.69	4	4	.....	.....	IV	3510.53	8	3	2	.....	III
3108.87	5	5	2	.....	III	3511.17	15	6	5	.....	III
3111.79	15	12	10	.....	III	3512.88	20	12	12	tr	III
3112.02	18	.....	.....	.....	V E	3521.55	6	5	3	1	III
3114.28	2	1	tr	.....	III	3549.02	150	10	6	.....	III E
3128.73	5	.....	.....	.....	V E	3551.78	8	?	?	.....	III ?
3129.94	10	.....	.....	.....	V E	3552.70	40	15 ?	15 ?	15	I
3135.18	25	.....	.....	.....	V E	3558.75	5	8 ?	6 ?	4	II
3152.66	15n	10	4	.....	III	3571.45	10	8	8	4	II
3155.62	3	12	8	.....	III A	3576.05	12	5	3	.....	III
3171.72	4	8	4	.....	III A	3584.53	15c	5 ?	?	.....	III ? E
3172.85*	4	15	12	tr	III A	3587.74	20	10	8	.....	III
3173.07	10	.....	.....	.....	V E	3589.66	5 ?	?	?	.....	III ?
3179.43	40	.....	.....	.....	V E	3592.90	200	100R	50r	25	II
3185.93	2	10	8	.....	III A	3600.77	500	20	15	1	III E
3191.28	15	15	12	.....	III	3601.96	200	10	8	.....	III E
3194.37	6	8	6	.....	III	3611.05	400	15	12	.....	III E
3195.62	100	15	10	.....	III E	3620.95	400	200R	75r	30	II
3200.27	100	15	10	.....	III E	3628.73	150	5	3	.....	III E
3203.33	100	15	10	.....	III E	3633.13	300	10	8	.....	III E
3209.38	3	10	10	.....	III A	3664.63	200	8	6	.....	III E
3216.70	150	20	15	.....	III E	3668.46	3	.....	.....	.....	V E
3242.29	200	25	20	.....	III E	3692.57	8	2	1	.....	III
3252.27	8	8	6	.....	III	3710.32	800	20	15	1	III E
3275.56	2	2	tr	.....	IV	3718.14	5	2	.....	.....	IV
3278.45	5	12	8	.....	III A	3738.60	4	3	1	.....	III

TABLE I—*Continued*

λ (I.A.)	Arc Int.	Furnace Intensities			Class	λ (I.A.)	Arc Int.	Furnace Intensities			Class
		High Temp.	Medium Temp.	Low Temp.				High Temp.	Medium Temp.	Low Temp.	
3747.57	20	3	1	.....	III E	4366.03	10	8	6	.....	III
3774.37	200	10	8	tr	III E	4370.98	1	1	1	.....	III
3776.57	40	5	3	.....	III E	4374.95	300	15	10	.....	III E
3788.73	100	6	4	.....	III E	4375.61	8	4	3	.....	III
3818.33	20	3	2	.....	III E	4379.33	6	8	6	.....	III
3832.94	30	4	3	.....	III E	4385.49	3	4	2	.....	III
3847.87	15	?	?	.....	V(?)E	4387.71	8	3	1	.....	III
3878.33	20	?	?	.....	V(?)E	4394.03	3	4	2	.....	III
3887.75	5	4	2	tr	III	4394.68	2	1	1	.....	III
3904.58	5	2	2	tr	III	4397.79	3	2	1	.....	III
3918.25	4	2	1	.....	III	4308.04	100	5	3	.....	III E
3930.66	10	.....	.....	.....	V E	4415.37	3	2	1	.....	III
3950.37	150	8	5	tr	III E	4417.44	2	3	1	.....	III
3951.61	8	.....	.....	.....	V E	4421.73	1	1	.....	.....	IV
3955.10	5	4	3	tr	III	4422.63	80	3	1	.....	III E
3982.64	100	6	4	tr	III E	4432.98	1	1	.....	.....	IV
3987.46	4	4	3	.....	III	4437.32	10	10	6	.....	III
4029.84	3	1	.....	.....	IV	4443.66	15	15	10	.....	III
4039.84	60	6or	30	20	II	4445.25	3	1	.....	.....	IV
4047.66	80	80R	40	25	II	4446.63	15	15	15	1	III
4077.39	300	300R	100R	60	II	4465.29	1	1	.....	.....	IV
4080.90	5	2	1	.....	III	4472.77	2n	tr	.....	.....	IV
4081.22	9	6	5	3	II	4473.89	7	3	1	.....	III
4083.72	100	100R	40	30	II	4475.70	20	12	10	1	III
4102.42	350	350R	150R	75	II	4476.92	25	15	12	2	III
4110.82	5	4	2	.....	III	4477.41	25	20r	15	2	III
4124.95	20	3	3	tr	III E	4479.01	2	1	.....	.....	IV
4128.35	300	300R	125R	70	II	4484.45*	5	8	8	3?	II?
4142.88	200	200R	100r	60	II	4487.29	20	15	10	1	III
4157.64	6	4	2	.....	III	4487.48	40	20	20	4	III
4167.57	100	100R	50	30	II	4491.75	5	4	4	.....	III
4174.18	100	100R	50	30	II	4492.43	4	3	3	tr?	III
4177.58	125	6	4	tr	III E	4505.95	50	25	25	6	II
4199.26	8	.....	.....	.....	V E	4513.57	4	6	6	1	III
4204.72	20	1	.....	.....	V E	4514.02	8	4	2	.....	III
4213.02	6	2	.....	.....	IV	4522.08	3	2	1	.....	III
4213.54	5	4?	2?	.....	III?	4527.25	80	40	40	15	II
4217.82	10	6	4	.....	III	4527.81	50	30	30	6	II
4220.62	30	12	6	.....	III	4528.1	3	1	tr	.....	III
4224.27	4	6	4	1	III	4533.53	1	1	.....	.....	IV
4229.20	2	1	1	.....	III	4534.12	3	1	tr	.....	III
4232.56	1	1	tr	.....	III	4537.21	2	2	1	.....	III
4235.73	40	2	tr	.....	IV E	4539.61	1	1	1	.....	III
4235.94	100	100R	40	30	II	4544.31	10	5	3	.....	III
4249.87	1	1	tr	.....	III	4554.47	2	1	1	.....	III
4250.37	1	1	1	.....	III	4559.37	10	7	5	1	III
4251.17	40	10	8	tr	III	4564.41	4	4	3	.....	III
4266.89	1	1	tr	.....	III	4564.94	2	1	—?	.....	IV?
4272.14	2	1	tr	.....	III	4570.71	3n	2	1	.....	III
4274.18	1	10	8	2	III A	4573.59	10	4	2	.....	III
4291.04	2	2	1	.....	III	4578.86	1	1	tr	.....	III
4300.37	2	1	tr	.....	III	4581.33	6	15	12	4	II A
4302.29	50	12	10	1	III	4581.81	3	10	8	1	III A
4309.63	125	8	5	.....	III E	4582.18	2	1	1	.....	III
4315.52	2	1	tr	.....	III	4590.77	1	.....	.....	.....	V
4316.32	3	3	3	.....	III	4594.00*	15	10	8	6?	II?
4322.32	2	2	3	.....	III	4596.56	12	5	3	.....	III
4329.93	1	1	tr	.....	III	4601.32	2	1	tr	.....	III
4330.76	10	10	8	1	III	4604.78	10	4	4	.....	III
4337.27	2	1	1	.....	III	4613.00	4	2	1	.....	III
4344.64	4	2	1	.....	III	4636.50	1	1	.....	.....	IV
4346.16	1	1	1	.....	III	4643.68	150	100R	80	80	I
4348.77	60	15	12	1	III	4653.79	4	1	tr	.....	III
4352.33	5	4	3	.....	III	4658.31	12	6	3	.....	III
4352.68	4	1	1	.....	III	4658.89	6	2	2	.....	III
4357.70	10	4	2	.....	III	4666.39	3	1	1	.....	III
4358.69	50	6	4	tr	III E	4666.85	3	1	1	.....	III

TABLE I—*Continued*

λ (I.A.)	Arc Int.	Furnace Intensities			Class	λ (I.A.)	Arc Int.	Furnace Intensities			Class
		High Temp.	Medium Temp.	Low Temp.				High Temp.	Medium Temp.	Low Temp.	
4667.47	8	4	2	.....	III	5380.65	15	12	4	1	III
4670.83	4	8	3	.....	III A	5402.81	20	.....	.....	.....	V E
4674.85	125	70R	60	60	I	5417.10	3	4?	.....	.....	IV?
4678.34	3	1	tr	.....	III	5424.40	20	10	3	tr	III
4682.31	20	.....	.....	.....	V E	5438.25	50	30	8	4	II
4689.76	4	3	2	.....	III	5466.51	300	150	100	50	II
4691.94	1	2	1	.....	III A	5468.48	40	10	3	.....	III
4696.79	8	5	5	tr	III	5473.40	10	.....	.....	.....	V E
4699.24	1	?	?	.....	III?	5480.75	8	.....	.....	.....	V E
4701.02	2	?	1	.....	III	5491.48	1	.....	.....	.....	V
4708.87	3	?	?	.....	III?	5493.23	15	12	2	.....	III
4725.84	3	1	tr	.....	III	5495.63	8	5	1	.....	III
4728.50	20	8	?	?	III?	5497.43	25	.....	.....	.....	V E
4732.38	3	2	1?	.....	III?	5503.47d	100	5	2	.....	III
4741.42	8	4	2	.....	III			25	8	4	II
4752.81	12	8	5	tr	III	5509.94	40	3?	.....	.....	V E
4761.02	40	30	30	30	I	5513.66	20	10	3	1	II
4780.17	1	1	1	.....	III	5521.65*	30	15	5	2	II
4781.06	10	12	10	2	III	5526.73	2	2	.....	.....	IV
4786.61	10	1	tr	.....	IV E	5527.59	250	150	80	50	II
4786.90	10	8	8	.....	III	5541.05	3	3	1	.....	III
4799.31	15	12	10	3	III	5544.60*	25	8	2	.....	III
4804.31	4	2	1	.....	III	5546.03	3	.....	.....	.....	V E
4804.81	6	3	2	.....	III	5556.48	20	100	200	200	I A
4819.63	10	10	8	2	III	5567.76	15	6	2	1	II
4821.62	2	1	.....	.....	IV	5577.43	30	20	6	2	II
4822.09	8	6	3	.....	III	5581.09	3	1	.....	.....	IV
4823.28	12	.....	.....	.....	V E	5581.89	150	150	80*	50	II
4839.15	3	3	2	.....	III	5590.96	2	2?	tr	.....	IV
4839.84	60	20	20	10	II	5594.13	2	2	tr	.....	IV
4845.67	50	15	15	8	II	5606.35	20	15	8	5	II
4852.67	50	15	15	8	II	5630.14	100	75	50	35	II
4854.25	4	4	2	.....	III	5632.25	3	8	10	10	I A
4854.89	70	1	.....	.....	V E	5644.70	20	12	8	6	II
4856.72	3	3	2	.....	III	5648.49	20	7	3	2	II
4859.82	40	15	15	6	II	5662.95	50	tr	.....	.....	V E
4863.11	1	1	tr	.....	III	5669.23	1	1	.....	.....	IV
4879.64	4	4	2	.....	III	5675.30	15	10	6	3	II
4883.71	80	1	.....	.....	V E	5675.64	1	.....	.....	.....	V
4886.30	4	4	2	.....	III	5686.68	1	1?	tr?	.....	IV?
4886.65	3	3	1	.....	III	5693.64	2	5	4	2	II A
4893.42	6	12	5	.....	III A	5706.76	30	15	12	3	II
4900.13	80	20	10	2	III E	5713.85	3n	1	.....	.....	IV
4906.11	6	12	6	.....	III A	5714.92	3n	1	.....	.....	IV
4909.00	2	3	1	.....	III	5720.61	3	3	tr	.....	IV
4912.02	tr	?	tr	.....	III A?	5723.45	1	1	.....	.....	IV
4921.88	10	20	10	2	III A	5728.91	3	.....	.....	.....	V E
4926.32	tr	1	tr	.....	III A	5732.08	3	4	3	2	II
4928.21	1	1	tr	.....	III	5740.21	1	3	1	.....	III A
4930.95	2	3	1	.....	III	5743.87	15	6	2	tr	III
4948.58	tr	tr	.....	.....	IV	5765.67	12	4	1	.....	III
4950.68	2	3	1	.....	III	5773.94	4	2	tr	.....	IV
4974.31	10	4	?	.....	III?	5781.69	2	.....	.....	.....	V E
4982.12	8	.....	.....	.....	V E	5787.74	1	tr	.....	.....	IV
5006.98	10	4	.....	.....	IV	5797.13	1	2	tr	.....	IV A
5070.18	5	.....	.....	.....	V	5821.88	4	4	1	.....	III
5072.17	5	1	.....	.....	IV	5832.26	3	1	.....	.....	IV
5087.43	150	4	.....	.....	V E	5871.76	2	1	.....	.....	IV
5088.18	3	.....	?	.....	V	5879.95	3	1	.....	.....	IV
5119.12	15	.....	.....	.....	V E	5902.94	7	3	.....	.....	IV
5123.21	40	?	.....	.....	V E	5945.71	6	3?	—?	.....	IV
5135.22*	25	10?	6?	—?	III	5950.00	5	2	.....	.....	IV
5196.44	2	.....	.....	.....	V E	6009.20	10?	8	1	.....	IV
5200.43	60	3	.....	.....	V E	6023.43	20?	200	200	200	I A
5205.73	80	5	.....	.....	V E	6024.33	?	4	1	.....	III
5240.80	15	8	tr	.....	IV	6135.06	?	4	1	.....	III
5289.82	4	.....	.....	.....	V E	6138.42	15?	150	150	150	I A

TABLE I—*Continued*

λ (I.A.)	Arc Int.	Furnace Intensities			Class	λ (I.A.)	Arc Int.	Furnace Intensities			Class
		High Temp.	Medium Temp.	Low Temp.				High Temp.	Medium Temp.	Low Temp.	
6191.72.....	100?	400	400	400	I A	6815.15.....	5	2	.....	.....	IV
6222.57.....	50?	250	250	250	I A	6832.47.....	1	.....	.....	.....	V E
6338.15.....	5	6	10	8	I	6845.23.....	10	2	1	.....	III
6402.02.....	50	200	200	150	I A	6858.22.....	1	.....	.....	.....	V E
6435.03	500	500	500	500	I	6887.21.....	8	1	.....	.....	IV
6437.17.....	5	1?	.....	.....	IV?	6895.99.....	3	.....	.....	.....	V E
6462.58.....	8	3	1	.....	III	6908.27.....	2	2	1	.....	III
6493.77.....	4	.....	.....	.....	V	6933.53.....	6	20	15	5	II A
6538.57.....	35	10	3	—?	III	6950.31.....	8	1	.....	.....	IV
6557.36.....	30	200	200	150	I A	6951.66.....	1	.....	.....	.....	V E
6576.86	6	3	1	.....	III	6958.01*....	4	100	125	100	I A
6584.88.....	5	40	30	20	II A	6979.86.....	8	—?	.....	.....	V?
6613.74.....	15	.....	.....	.....	V E	7008.95.....	5	.....	.....	.....	V
6622.51.....	2	1?	.....	.....	IV?	7009.95.....	4	.....	.....	.....	V
6636.48.....	2	1?	.....	.....	IV?	7035.19.....	5	1	.....	.....	IV
6650.60.....	8	2	.....	.....	IV	7052.97.....	10	4	2	.....	III
6664.38.....	3	1?	.....	.....	IV?	7054.32.....	4	.....	.....	.....	V
6687.58.....	80	100	100	100	I	7127.93.....	2	2?	1?	.....	III?
6691.85.....	2	.....	.....	.....	V	7191.63.....	10	3	1	.....	III
6694.75.....	1	.....	.....	.....	V	7264.19.....	8	2	.....	.....	IV E
6699.26.....	3	1	.....	.....	IV	7346.44.....	10	4	2	.....	III
6700.71.....	20	5	2	.....	III	7450.29†....	5	2	1?	.....	III(?)E
6713.20.....	6	2	1	.....	III	7563.06.....	5	.....	.....	.....	V
6735.99.....	7	2	1	.....	III	7719.89.....	2	.....	.....	.....	V
6793.69.....	80	100	100	100	I	7724.01.....	2	1	.....	.....	IV
6795.40.....	15	.....	.....	.....	V E	7881.87.....	10	.....	.....	.....	V E
6803.15.....	1	.....	.....	.....	V						

NOTES—YTTRIUM

3018.93 Blend *Fe* in arc.
3056.33 Blend *V*. Probable intensity of *V*-line subtracted.
3172.85 May be impurity.
3461.02 Blend *Dy* in furnace.
3471.77 Furnace line may be *Er*.
4484.45 May be impurity.
4594.00 Furnace line may be partly impurity.
5135.22 Blend with impurity line.
5521.65 Blend enhanced line.
5544.60 Blend enhanced line.
6958.01 Probably impurity, not *La*.

The limited quantity of these preparations permitted their use, as a rule, only in the arc. For the furnace, which required frequent recharging of the tube during long runs, fairly pure yttrium nitrate and lanthanum chloride were obtained from Eimer and Amend; and, by using only lines which were given by the purer arc material, it is hoped that foreign lines have been largely eliminated. For the zirconium spectrograms, the metal in pulverized or flake form proved satisfactory.

EXPLANATION OF THE TABLES

Tables I, II, and IV present the intensities in arc and furnace and the resulting classification for lines of yttrium, zirconium, and

TABLE II

Temperature Classification of Zirconium Lines

λ (I.A.)	Arc Int.	Furnace Intensities		Class	λ (I.A.)	Arc Int.	Furnace Intensities		Class
		High Temp.	Low Temp.				High Temp.	Low Temp.	
2792.04	20	8	.....	IV	3233.11	5	.....	.....	V
2793.40	8	.....	.....	V	3234.13	100	100	40	II
2795.13	8	.....	.....	V	3234.80	5	4	.....	III
2806.78	12	4	.....	IV	3241.05	40	2	.....	IV E
2814.91	150	12	.....	IV	3243.98	8	.....	.....	V
2815.49	2	.....	.....	V	3250.39*	35	25	4	III
2819.56	10	4	.....	IV	3254.28	15	1	.....	IV
2821.57	8	3	.....	IV	3260.11	40	30	5	III
2827.55*	8	2	.....	IV	3269.66	40	30	5	III
2829.81	8	1	.....	IV	3272.22	40	1	.....	V E
2836.50	4	.....	.....	V	3273.05	150	5	.....	IV E
2837.23	200	20	.....	IV	3279.27	125	4	.....	IV E
2848.52	150	15	.....	IV	3282.73	15	20	6	II
2857.98	2	.....	.....	V	3284.72	80	3	.....	IV E
2860.85	30	6	.....	IV	3305.15	40	1	.....	V E
2866.38	2	.....	.....	V	3306.27	80	3	.....	IV E
2875.99	200	20	.....	IV	3314.50	12	1	.....	IV E
2880.84	5	.....	.....	V	3316.21	4	.....	.....	V
2892.27	30	8	.....	IV	3322.29	5	.....	.....	V
2898.26	3	.....	.....	V	3323.00	20	1	.....	IV E
2916.25	10	4	.....	IV	3333.54	4	1?	.....	IV?
2916.49	2	.....	.....	V	3353.05	20	20	6	II
2923.85	10	4	.....	IV	3356.09	60	1	.....	V E
2940.77	3	.....	.....	V	3357.27	50	1	.....	V E
2960.87	60	20	.....	IV	3360.45	25	15	3	II
2964.85	2	.....	.....	V	3368.61	5	4	1	II
2969.19	40	15	.....	IV	3370.71	20	8	1	III
2985.39	200	30	6	II	3379.92	10	3	1	III
3005.37	40	8	.....	IV	3385.78	5	.....	.....	V
3005.50	60	6	.....	IV	3387.87	40	1	tr	III E
3011.75	250	40	8	II	3388.30	80	2	1	III E
3014.44	15	10	.....	III	3391.98	400	6	2	III E
3029.52	300	50	12	II	3393.12	25	tr	.....	V E
3043.26	3	3	.....	III	3397.92	5	.....	.....	V
3045.83	10	8	.....	III	3401.78	3	.....	.....	V
3049.35	6	6	.....	III	3414.66*	20	8	3	II
3063.59*	8	4	.....	III	3419.66	20	10	4	II
3072.88	3	.....	.....	V	3430.29	8	10	4	II
3085.35	12	12	.....	III	3430.53	50	1	.....	V E
3090.44	6	5	.....	III	3438.24	400	12	3	III E
3092.29*	2?	.....	.....	V	3440.44	30	30	15	II
3093.34*	2?	2	.....	III	3446.61	30	15	6	II
3094.80	15	15	.....	III	3447.37	100	50	40	II
3095.82	12	10	.....	III	3449.91	6	2	.....	IV
3099.69	2	.....	.....	V	3455.91	30	15	5	II
3108.36	10	8	.....	III	3457.19	20	10	3	II
3113.50	8	6	.....	III	3461.09	30	7	2	II
3120.75	125	40	4	III	3465.63	10	10	4	II
3125.92	40	1	.....	V E	3469.10	4	4	2	II
3129.77	40	1	.....	V E	3471.19*	100	50	40	II
3130.06	8	5	.....	III	3472.89	4	1	.....	IV
3131.11	25	15	.....	III	3478.79	20	10	.....	III
3132.06	50	40	3	III	3482.81	20	20	1	III
3133.24	20	10	.....	III	3483.01	15	2	.....	IV
3136.96	30	15	tr	III	3501.35	15	15	.....	III
3138.68	50	1	.....	V E	3501.50	8	1	.....	IV
3139.80	20	12	.....	III	3509.33	100	100	15	II
3148.81	25	15	.....	III	3519.61	125	125	30	II
3157.82	50	40	3	III	3530.23	15	10	1	III
3165.98	80	3	.....	IV E	3533.23	60	50	3	III
3191.22	100	60	20	II	3535.17	20	4	.....	IV
3191.91	10	tr?	.....	V E	3547.69	100	60	15	II
3204.90	12	6	.....	III	3549.74†	15	10?	2?	III?
3212.02	100	80	30	II	3550.47	30	30	4	III
3212.58	8	3	.....	III	3558.96†	12	?	.....	IV?
3231.70	70	4	.....	IV E	3566.11	100	100	40	II

TABLE II—*Continued*

λ (I.A.)	Arc Int.	Furnace Intensities		Class	λ (I.A.)	Arc Int.	Furnace Intensities		Class
		High Temp.	Low Temp.				High Temp.	Low Temp.	
3568.87†	15	3?	.....	IV?	4024.92	40	15	3	II
3575.80	100	50	10	II	4027.20	40	15	5	II
3577.56†	15	?	.....	IV?	4028.95	15	8	1	III
3586.30	100	50	10	II	4030.04	30	15	3	II
3591.73	2	1	tr	III	4030.75*	3	−?	.....	IV?
3601.20	400	200	60	II	4035.89*	40	20	8	II
3613.70	15	10	.....	III	4041.64†	5	−?	.....	IV
3623.87	300	150	30	II	4042.22	10	2	.....	IV
3634.15	50	50	4	III	4043.58	30	15	6	II
3638.69	3	tr	.....	IV	4044.56	25	12	3	II
3656.89	6	3	.....	III	4050.48	10	5	2	II
3661.20	30	20	3	III	4054.43	10	7	3	II
3663.65	300	150	30	II	4055.04	60	30	10	II
3680.38	5	2	.....	III	4055.71†	20	1?	.....	V?
3706.63	10	5	tr	III	4058.63†	3	1?	.....	IV?
3714.13	30	20	2	III	4060.08†	2	?	.....	IV?
3737.40	5	1	.....	IV	4060.58†	4	2?	.....	IV?
3764.39	80	50	12	II	4061.53	15	8	2	II
3766.72	60	40	8	II	4064.16	100	50	12	II
3780.54	100	60	15	II	4068.73*	4	2	1	II
3791.40	80	40	10	II	4072.71	100	50	15	II
3822.42	40	20	5	II	4074.93	15	8	2	II
3835.97	100	50	15	II	4076.53	10	5	1	III
3847.01†	30	?	?	?	4078.31	12	7	2	II
3849.26	30	15	4	II	4081.22	100	60	20	II
3863.88	100	30	10	II	4082.29	3	2	1	II
3864.34†	40	?	?	?	4083.09	4	2	1	II
3877.60	40	5	.....	IV	4084.30†	4	−?	.....	IV?
3879.05*	10	20	10	I A	4087.69	3	2	1	II
3885.43	100	80	20	II	4090.79	5	.....	.....	V
3890.32	125	80	25	II	4094.27†	3	−?	1	II?
3891.39	100	80	20	II	4099.31	4	4	1	III
3892.03	4n	tr	.....	IV	4107.50	8	.....	.....	V
3896.53	10	10	2	II	4108.40	15	5	1	III
3897.66	3	.....	.....	V	4121.46	30	10	2	III
3916.64	10	10	2	II	4135.68	10	4	2	II
3921.80	100	50	10	II	4152.64†	10	−?	.....	IV?
3926.79	3n	.....	.....	V	4166.45	20	10	3	II
3929.54	150	80	20	II	4183.32	10	6	.....	III
3933.19	2	.....	.....	V	4187.57†	20	10	−?	III
3941.63	5	1	.....	IV	4194.76	10	.....	.....	V
3963.80	2n	.....	.....	V	4199.09†	20	10?	?	III?
3966.67	50	30	6	II	4201.45	20	10	2	III
3968.26	80	50	15	II	4213.86†	15	8?	?	III?
3972.30	3	.....	.....	V	4218.44	6	4	tr	III
3973.51	60	40	10	II	4220.65	2	1	.....	III
3975.30†	15	2?	.....	IV	4222.45	5n	.....	.....	V
3977.33	3	.....	.....	V	4224.69	2n	.....	.....	V
3977.48	15	5	1	III	4227.76	200	100	40	II
3978.26	2	.....	.....	V	4234.63	7	5	2	II
3978.74	30n	4	1	III	4236.07	6	2	tr	III
3981.59	20n	2	.....	IV	4237.43	4	1	.....	IV
3982.16	5	.....	.....	V	4239.31	150	80	30	II
3984.75*	20	5	1	III	4240.34	50	25	10	II
3986.80	5	.....	.....	V	4241.20*	50	15?	?	II?
3988.68	10	3	.....	IV	4241.69	80	60	30	II
3989.29	7	6	1	III	4242.02	2	1	.....	IV
3989.50	12	.....	.....	V	4253.57	4	1	.....	IV
4001.09†	4	2?	.....	IV?	4256.04	1	1	.....	IV
4001.22	2	.....	.....	V	4256.44	4	3	tr	III
4002.55†	8	?	.....	IV?	4261.21	4	2	.....	III
4003.10†	12	?	.....	IV?	4261.43	3	.....	.....	V
4004.40†	4	?	.....	IV?	4268.02	20	10	6	II
4004.87†	4n	?	.....	IV?	4274.76	5	4	1	III
4007.60†	15	2?	.....	IV?	4276.72	5	4	.....	III
4012.25†	12	?	.....	IV?	4282.20	20	10	7	II
4017.00†	8	?	.....	IV	4291.20	5	3	1	II
4023.98	30	10	3	II	4291.35	8	4	1	III

TABLE II—*Continued*

λ (I.A.)	Arc Int.	Furnace Intensities		Class	λ (I.A.)	Arc Int.	Furnace Intensities		Class
		High Temp.	Low Temp.				High Temp.	Low Temp.	
4294.45	2	tr	.....	IV	4667.14	4	2	1	II
4294.80	20	10	7	II	4683.42*	15	—?	.....	V?
4302.88	15	7	3	II	4684.25†	2	?	.....	V?
4304.69	8	4	1	III	4687.80	150	20	20	I
4309.82	3	1	.....	IV	4688.45	40	15	20	I
4319.05	3	1	.....	IV	4707.78	4	.....	.....	V
4321.17	4	3	2	II	4710.08	100	20	15	I
4324.04	4	1	.....	IV	4711.91	8	.....	.....	V
4325.44	15	5	2	II	4713.43†	2	?	.....	IV?
4329.57	3	2	1	II	4717.62†	6	1?	.....	IV?
4341.13	20	5	2	II	4719.11	10	tr	.....	V
4343.04	4	3	2	II	4732.33†	15	4?	2?	III?
4346.52	4	1	.....	IV	4739.48	100	20	15	I
4347.23	2	1	.....	IV	4751.91*	2	1?	.....	IV?
4347.90	30	15	8	II	4753.05	3	.....	.....	V
4348.94	2	1	.....	IV	4762.77	8	1	.....	IV
4358.75	4	2	1	II	4772.32	80	15	12	I
4360.81	15	8	5	II	4784.92	12	5	3	II
4366.45	15	2	1	II	4788.68	8	3	2	II
4373.07	2	.....	.....	V	4789.12	2	1	.....	III
4394.50	2	2	1	II	4805.87	15	6	3	II
4394.94	8	1	.....	IV	4809.47	8	1	.....	IV
4395.20*	6	?	?	?	4815.05	12	5	3	II
4400.24	5	2	1	II	4815.63	60	12	10	I
4402.96	4	2	1	II	4824.29	12	5	3	II
4413.04	12	2	1	II	4828.04	10	4	2	II
4414.14	2	2	1	II	4838.78	4	1	.....	IV
4420.46	12	2	tr	III	4838.98	2	.....	.....	V
4429.10	3	2	.....	III	4851.37	12	5	2	II
4431.48	10	2	tr	III	4866.07	5	2	1	II
4438.05	8	3	1	II	4881.25	4	2	tr	III
4444.34	2	.....	.....	V	4883.60	5	2	1	II
4448.95	2	1	.....	IV	4893.12	2	tr	.....	IV
4450.28	5	1	.....	IV	4933.64	3	.....	.....	V
4455.44	2	1	.....	IV	4948.76	2	.....	.....	V
4456.30	3	tr	.....	IV	4994.76†	5	2?	.....	IV?
4459.02*	2	2	1	II	4996.33	2	.....	.....	V
4466.91	10	3	1	II	5046.58	10	.....	.....	V
4468.78	3	2	1	II	5060.39	3	.....	.....	V
4470.32	5	1	.....	IV	5064.90	15	.....	.....	V
4470.56	15	2	tr	III	5065.21	4	.....	.....	V
4480.77	2	.....	.....	V	5070.25	3	.....	.....	V
4482.50	2	.....	.....	V	5073.98	2	.....	.....	V
4491.56	2	1	.....	IV	5078.25†	20	?	?	V?
4494.93*	2	1?	.....	IV?	5085.26	3	.....	.....	V
4507.12	30	10	6	II	5115.24	5	.....	.....	V
4535.75	30	10	6	II	5133.39	6	.....	.....	V
4539.98	4	2	1	II	5155.45	12	.....	.....	V
4542.22	20	8	4	II	5158.00	7	.....	.....	V
4553.01	12	4	2	II	5160.99	3	.....	.....	V
4555.13	10	.....	.....	V	5165.94	4	.....	.....	V
4555.53	8	.....	.....	V	5183.70	6	4	.....	III
4558.04	2	.....	.....	V	5201.15	4	4	.....	III
4565.46	2	.....	.....	V	5209.30	4	.....	.....	V
4575.51	40	15	12	I	5224.92	6	6	tr	III
4576.21	2	.....	.....	V	5277.39	8	8	1	III
4582.29	3	.....	.....	V	5280.04	2	.....	.....	V
4584.24†	2	1?	.....	IV?	5294.82	2	.....	.....	V
4590.54	6	2	1	II	5296.79	5	5	1	III
4602.57	15	2	.....	IV	5311.40	6	6	1	III
4604.42	8	4	1	III	5362.54	6	4	.....	III
4610.10†	2	1?	.....	IV?	5385.12	40	15	4	II
4613.35†	1	1?	.....	IV?	5405.11	2	2	.....	III
4626.41†	12	2?	.....	IV?	5407.61†	7	5?	2?	II?
4634.02	50	20	20	I	5437.74†	2	?	.....	IV?
4640.12	2	.....	.....	V	5448.54	3	2	.....	III
4644.83	8	.....	.....	V	5478.31†	3	2?	.....	III?
4657.64	7	.....	.....	V	5480.81†	2	1?	.....	IV?

TABLE II—*Continued*

λ (I.A.)	Arc Int.	Furnace Intensities		Class
		High Temp.	Low Temp.	
5486.08	2	1	.....	IV
5502.13	15	1	.....	V
5517.10	2	.....	.....	V
5528.41†	8	—?	.....	V?
5537.41	2	.....	.....	V
5545.31	3	.....	.....	V
5620.13†	15	2?	.....	IV?
5623.52	3	.....	.....	V
5664.50	25	15	3	II
5680.89	15	8	1	II
5708.85†	2	2?	.....	III?
5735.68	20	20	2	II
5797.74	25	30	5	II
5869.48†	4	2?	.....	IV?
5879.77	40	40	8	II
5885.59	8	8	1	II
5901.03†	2	2?	.....	III
5925.12†	12	4?	1?	III?
5935.18	10	12	3	II
5955.34	12	15	3	II
5984.22	5	.....	.....	V
6032.60†	5	2?	.....	IV?
6045.88†	4	?	?	IV?
6049.24†	3	1?	.....	IV?
6062.88	12	10	2	II
6121.87†	3	?	.....	IV?
6121.96†	20	15	?	II?
6124.87†	5	5?	.....	III?
6127.49	200	200	50	II
6134.58	125	125	30	II
6140.50	15	10	3	II
6143.23	150	150	40	II
6157.75†	20	2?	.....	IV?
6192.96†	5	5?	?	III?
6213.07†	8	3?	.....	IV?
6214.73	6	.....	.....	V
6257.28	8	.....	.....	V
6299.73†	12	—?	—?	V?
6313.05†	50	—?	.....	V
6321.38	2	.....	.....	V
6340.39	1	.....	.....	V
6345.23†	8	5?	.....	IV?
6346.54	2	.....	.....	VE
6407.04	4	.....	.....	V
6434.41	1	.....	.....	V
6445.75	10	3	.....	IV
6457.55	1	.....	.....	V
6470.25	15	2	.....	IV
6489.68	25	3	.....	IV
6493.13†	1	—?	.....	V?
6503.31†	5	2?	.....	IV?
6506.38†	6	3?	.....	IV?
6550.54	3	.....	.....	V
6569.43	1	.....	.....	V
6576.58	1	.....	.....	V
6592.02	4	.....	.....	V
6603.27	4	.....	.....	V
6620.64	1	.....	.....	V
6688.19	2	.....	.....	V
6702.14	1	.....	.....	V
6709.62†	2	2?	.....	IV?
6717.90†	3	3?	1?	III?
6752.73	4	.....	.....	V
6762.39	30	8	1	III
6769.17	40	2	.....	V
6772.94	3	.....	.....	V
6790.90	4	.....	.....	V
6828.83	10	.....	.....	V
6832.93†	12	5?	1?	III?
6847.06	25	1	.....	V
6849.30	4	.....	.....	V
6888.31	20	2	.....	IV
6900.60	2	1	.....	IV
6904.40	2	3	.....	III
6907.41	3	1	.....	IV
6916.97	2	2	.....	III
6932.41	2	.....	.....	V
6948.51	4	.....	.....	V
6953.87†	60	10?	1?	III?
6966.49	15	2	1	III
6990.86	50	6	1	III
6994.39†	15	?	?	IV?
7027.41	30	5	.....	III
7057.39	3	.....	.....	V
7057.99	1	.....	.....	V
7087.36	20	5	.....	III
7089.51	1	.....	.....	V
7094.03	3	1	.....	IV
7095.67	5	.....	.....	V
7097.79	150	30	5	II
7102.95	80	25	4	II
7103.77†	40	—?	—?	II?
7111.71	20	5	1	II
7112.83†	4	?	?	IV?
7113.55	1	.....	.....	V
7169.15	150	50	20	II
7201.68	4	.....	.....	V
7264.80	4	.....	.....	V
7306.24	1	.....	.....	V
7311.69	2	.....	.....	V
7313.73	2	.....	.....	V
7318.16†	8	3?	1?	III?
7336.04	4	5	1	II
7344.00†	4	—?	?	III?
7373.52	1	.....	.....	V
7383.66	1	.....	.....	V
7400.90	tr	.....	.....	V
7439.89	10	5	.....	III
7467.59†	1	?	.....	V?
7502.91	1	.....	.....	V
7515.80	1	.....	.....	V
7540.66	1	.....	.....	V
7544.62	1	.....	.....	V
7551.50	2	.....	.....	V
7554.73	5	3	.....	III
7558.47	2	.....	.....	V
7607.19	8	3	.....	III
7621.17	1	.....	.....	V
7658.63†	3	?	.....	IV?
7690.82†	1	?	.....	V?
7819.36	5	.....	.....	V
7822.97	2	.....	.....	V
7826.76	2	.....	.....	V
7849.39	5	3	.....	III
7870.01†	4	4?	.....	III
7944.65	6	.....	.....	V
7956.69†	2	?	.....	IV
7960.02	2	.....	.....	V
8063.11	3	.....	.....	V
8070.12	20	5	.....	III
8133.00	15	5	.....	III
8194.67	20	.....	.....	V
8194.91		.....	.....	V
8212.59†•	10	4?	.....	III?
8305.94†•	8	2?	.....	III?

TABLE II—*Continued*

NOTES—ZIRCONIUM

2827.55 Double. Blend with enhanced line.
3063.59 Enhanced line on red side.
3092.29, 3093.34 In wing of *Al* line in arc.
3250.39 Blend enhanced line.
3414.66 Blend enhanced line.
3471.19 Blend enhanced line.
3879.05 Unusual type. Possibly impurity.
3984.75 Blend enhanced line.
4030.75 Blend *Mn*.
4035.89 Enhanced line on violet side.
4068.73 Blend with enhanced line λ 4068.63.
4241.20 Strong line shows here at low temperature, probably due to blend with low-temperature line of unknown origin.
4395.20 Blend *V* in furnace.
4494.93 May be impurity.
4683.42 Faint if present in furnace.
4751.91 May be band head in furnace.

lanthanum, respectively. Very faint lines which appear only in the arc are usually not included, unless they have about the same intensity as neighboring lines which appear also in the furnace, so that a comparison may be useful on this account. Enhanced lines are listed when they are of fair intensity in the arc, since many of the enhanced lines of yttrium and lanthanum appear in the furnace, and those which are absent are of interest as requiring a higher excitation than that of the furnace. The case is different with zirconium, where a large proportion of the arc spectrum consists of enhanced lines, very few of which appear in the furnace. To avoid unnecessary length in the furnace tables, these zirconium enhanced lines, with their arc intensities, are listed in Table III. Those which by reason of their presence in the furnace spectrum are classified in Table II are indicated by "f" after the wave-length. Lines not so indicated may be considered as belonging in class VE.

Wave-lengths.—The international system is used throughout. Wave-lengths shorter than λ 5500 for yttrium are taken largely from Exner and Haschek,[1] changed to I.A. Use was also made of the tables of Yntema and Hopkins,[2] and of Eder.[3] Zirconium wave-lengths for the same region are chiefly reduced from Vahle,[4] and those of lanthanum from Wolff.[5] Wave-lengths greater than λ 5500

[1] *Spektren der Elemente bei normalem Druck*, Leipzig, 1911.

[2] *Journal of the Optical Society of America*, **6**, 121, 1922.

[3] *Sitzungsberichte der Akademie der Wissenschaften in Wien*, IIa, **125**, 471, 1916.

[4] *Zeitschrift für wissenschaftliche Photographie*, **18**, 84, 1918.

[5] *Ibid.*, **3**, 395, 1905.

TABLE III

Zirconium Enhanced Lines

λ	Arc. Int.	λ	Arc. Int.	λ	Arc. Int.	λ	Arc. Int.
2630.87	40	2910.25	8	3165.47	10	3399.80	1
2639.09	30	2915.99	40	3165.98f	80	3402.88	6
2643.41	4	2916.63	8	3166.27	12	3403.69	10
2650.38	8	2918.26	30	3178.05	10	3404.84	50
2667.80	20	2924.64	2	3181.56	10	3408.06	8
2669.46	4	2925.63	2	3181.97	10	3410.27	60
2678.62	150	2926.99	20	3182.88	60	3413.36	4
2681.76	2	2934.60	20	3191.91f	10	3414.66*	6
2689.45	5	2936.30	20	3208.29	4	3419.10	6
2692.63	5	2937.74	1	3212.83	8	3424.81	10
2693.55	30	2945.45	3	3214.18	60	3430.53f	50
2694.06	20	2948.96	20	3222.52	5	3431.60	5
2695.45	3	2951.48	25	3228.82	15	3432.42	5
2699.61	15	2952.24	8	3231.70f	70	3433.91	4
2700.12	100	2955.78	15	3241.05f	40	3437.13	20
2711.50	50	2962.70	30	3242.16	2	3438.24f	400
2712.41	25	2964.56	1	3250.41	35	3443.58	3
2714.25	20	2968.96	30	3264.84	4	3454.56	3
2720.34	2	2969.64	20	3271.13	8	3457.56	30
2722.59	150	2976.61	8	3272.22f	40	3458.93	10
2726.50	125	2978.06	40	3273.05f	150	3463.01	30
2732.70	80	2979.18	20	3275.16	2	3469.95	2
2734.82	150	2980.85	25	3279.27f	125	3471.18	100
2740.36	4	2987.75	40	3281.71	2	3478.30	5
2740.50	8	2990.14	1	3282.84	4	3478.49	2
2741.55	15	2991.43	5	3284.72f	80	3479.02	10
2742.55	125	2998.48	1	3285.89	2	3479.39	60
2745.87	100	3003.76	20	3287.26	2	3480.40	5
2750.95	3	3009.89	2	3288.79	8	3481.15	80
2752.22	150	3012.89	1	3296.41	4	3483.53	30
2754.23	2	3013.33	8	3302.67	4	3496.23	200
2758.82	80	3019.84	15	3305.15f	40	3499.59	6
2761.91	5	3020.47	30 ?	3306.27	80	3505.51	10
2768.76	25	3025.17	1	3309.93	2	3505.68	20
2774.17	15	3025.61	1	3311.39	4	3510.45	4
2783.57	4	3028.05	20	3313.71	8	3525.80	10
2796.91	12	3030.92	15	3314.50f	12	3542.60	10
2799.15	5	3036.39	10	3318.53	6	3551.97	40
2808.17	2	3036.48	5	3319.04	15	3556.60	60
2810.94	15	3044.11	1	3323.00f	20	3572.45	100
2818.74	20	3048.26	1	3326.81	10	3573.09	8
2825.58	30	3048.39	1	3334.24	15	3576.80	40
2827.54*	8	3052.83	2	3334.61	15	3578.21	4
2833.94	6	3054.86	20	3337.93	1	3587.92	10
2834.43	3	3057.03	20	3338.41	12	3607.35	5
2838.04	4	3057.22	2 ?	3340.58	50	3611.90	8
2839.35	8	3060.12	4	3344.81	15	3613.11	40
2843.52	10	3061.34	4	3354.41	10	3614.77	40
2844.58	50	3064.73	3	3356.09f	60	3630.05	10
2848.16	15	3065.22	3	3357.27f	50	3636.42	5
2851.99	40	3086.45	1	3360.00	10	3668.40	5
2854.44	3	3089.02	1	3362.73	4	3671.25	10
2856.07	1	3095.10	25	3363.85	8	3674.69	40
2865.10	2	3099.27	25	3367.84	7	3697.48	10
2865.62	8	3106.60	40	3369.30	2	3698.17	40
2869.82	40	3110.88	20	3373.43	15	3709.26	30
2872.54	2	3111.18	3	3374.73	30	3714.77	6
2877.54	10	3115.73	1	3376.28	8	3731.25	10
2882.11	3	3125.22	5	3377.43	10	3751.64	30
2883.81	3	3125.92f	40	3378.33	4	3766.82	30
2888.04	6	3128.82	1	3382.92	3	3915.95	15
2889.42	3	3129.20	25	3387.87f	40	3934.12	15
2898.73	12	3129.77f	40	3388.30f	80	3934.78	10
2901.64	4	3133.49	20	3391.98f	400	3936.06	2
2901.84	6	3134.72	1	3393.12f	25	3958.19	25
2902.29	1	3138.68f	50	3394.56	2	3984.72	20
2904.31	1	3155.67	20	3396.34	8	3991.10	30
2905.24	25	3157.00	15	3396.64	2	3998.04	20
2907.40	5	3164.32	25	3399.36	40	4018.10	10

* Blend arc line.

TABLE III—*Continued*

λ	Arc. Int.	λ	Arc. Int.	λ	Arc. Int.	λ	Arc. Int.
4018.35	4	4208.97	20	4317.32	3	4494.42	2
4023.30	2	4211.90	10	4333.29	1	4495.48	1
4024.42	5	4224.25	2	4337.64	1	4496.96	15
4029.66	15	4230.17	1	4339.54	1	4553.96	1
4034.09	2	4231.63	2	4342.24	1	4574.48	1
4040.25	2	4236.55	3	4345.55	1	4613.98	2
4045.63	10	4258.06	10	4359.74	8	4629.07	1
4048.65	15	4264.89	1	4370.97	6	4661.81	1
4050.36	15	4266.74	5	4379.74	8	4685.22	1
4071.10	2	4273.53	2	4403.33	3	4703.07	1
4085.68	5	4277.36	1	4414.53	5	5112.27	5
4090.54	5	4286.51	1	4440.43	5	5191.58	15
4096.62	2	4293.15	1	4442.50	1	5311.76	1
4149.20	50	4296.76	1	4443.00	12	5350.07	6
4151.02	8	4301.84	1	4454.80	8	5350.34	6
4156.24	15	4308.95	1	4457.41	10		
4161.24	15	4312.24	1	4461.20	5		

are those of Kiess,[1] who kindly furnished an advance list of zirconium wave-lengths to supplement those published for yttrium and lanthanum.

Intensities and classes.—The intensity estimates are in accordance with the procedure followed in previous papers, a line distinctly outlined in the negative being given intensity "1." The large range of line-intensities found in these spectra requires high values for many of the stronger lines, and it is doubtful if high enough values have been assigned to some of the strongest. The relative strength of the ionized lines with respect to those of the neutral atom is controlled by arc conditions which vary for different exposures, so that a fair degree of consistency among the line-intensities for each set is all that can be attained.

Classes.—In the final column, the usual method of classification is followed. Lines in classes I and II appear at low temperature. Those of class I show a slower change from low to high temperature than those of class II, and as a rule are less conspicuous in the arc. Lines of class III are usually well developed at medium temperature, while lines clearly associated with high temperatures are placed in classes IV and V, those of class V being absent or very faint in the furnace spectrum.

For zirconium the initial temperature required for the appearance of lines in the furnace is so high that only two stages could be

[1] *Scientific Papers of the Bureau of Standards*, **17**, 317 (No. 421), 1921.

TABLE IV

Temperature Classification of Lanthanum Lines

λ (I.A.)	Arc Int.	Furnace Intensities			Class	λ (I.A.)	Arc Int.	Furnace Intensities			Class
		High Temp.	Medium Temp.	Low Temp.				High Temp.	Medium Temp.	Low Temp.	
2798.54...	1	.....	.....	.....	V E	3649.51...	40	30r	8	5	II
2808.36...	10	.....	.....	.....	V E	3650.18...	125	12	1	.....	IV E
2855.89...	1	.....	.....	.....	V E	3662.07...	50	5	.....	.....	IV E
2862.95...	1	.....	.....	.....	V E	3672.01...	30	25	10	2	III
2880.63...	3	.....	.....	.....	V E	3699.53...	12	20	8	.....	III A
2885.14...	4	.....	.....	.....	V E	3701.80...	4	.....	.....	.....	V E
2893.68...	4	.....	.....	.....	V E	3704.51...	40	30	10	5	II
2950.51...	2	.....	.....	.....	V E	3705.82d..	125	2	.....	.....	V E
3010.80*..	2	8	.....	.....	IV A	3713.55...	300	20	1	.....	IV E
3096.00...	10	20	5	4	II A	3714.86...	60	2	.....	.....	V E
3104.58...	60	10	.....	.....	IV E	3715.53...	80	8	tr	.....	IV E
3108.45...	5	1	.....	.....	IV E	3724.77...	2	.....	.....	.....	V
3109.42...	12	50	6	2	II A	3725.05...	40	4	.....	.....	IV E
3142.75...	50	8	.....	.....	IV E	3735.84...	20	2	.....	.....	IV E
3148.52...	4	20	3	tr	III A	3759.08...	600	25	8	.....	III E
3171.66*..	.....	.....	.....	.....	V E	3780.67...	50	1	.....	.....	V E
3175.96...	15	50	5	2	II A	3784.80...	20	1	.....	.....	V E
3179.76...	8	30	4	tr	III A	3790.82...	600	25	10	.....	III E
3193.00*..	{ 15?	3?	.....	.....	IV E	3794.78...	600	20	8	.....	III E
	{ 10?	2?	.....	.....	IV E	3835.09...	20	.....	.....	.....	V E
3215.80...	15	50	10	3	II A	3840.68†..	50	?	.....	.....	V (?) E
3235.63...	5	30	5	.....	III A	3846.00†..	20	?	— ?	.....	V (?) E
3245.12...	200	40	6	.....	III E	3849.02†..	200	?	?	.....	III (?) E
3247.03...	8	20	8	2	II A	3871.61†..	300	10?	6?	.....	III (?) E
3249.35...	100	20	6	.....	III E	3886.31...	400	12	4	.....	III E
3265.65d..	150	50	15	.....	III E	3895.65*..	8?	2?	.....	.....	IV
3303.10d..	200	70	20	.....	III E	3898.59...	40	25	20	10	II
3306.98...	10	1	.....	.....	IV E	3902.57...	20	20	20	6	II
3337.47...	500	50	15	.....	III E	3910.80*..	15	2?	.....	.....	IV (?) E
3342.22...	20	50	15	6	II A	3916.04d..	400	30	5	.....	III E
3344.54...	300	30	10	.....	III E	3921.53...	300	30	4	.....	III E
3349.81...	3	15	4	.....	III A	3927.56...	80	60r	40	40	I
3357.46...	7	20	10	1	III A	3929.21...	500	40	8	.....	III E
3362.03...	12	25	12	2	III A	3936.22...	60	8	1	.....	IV E
3376.31...	40	8	2	.....	III E	3949.09d..	1000	60	15	.....	III E
3380.88...	400	50	20	.....	III E	3953.68...	40	30	25	10	II
3381.42...	15	30	10	3	II A	3988.52d..	800	40	8	.....	III E
3388.60...	12	20	8	2	II A	3995.76...	600	40	8	.....	III E
3397.74...	4	.....	.....	.....	V E	4015.41...	50	50r	40	40	I
3404.51...	15	30	10	2	III A	4023.58d..	15	2	.....	.....	IV E
3450.65...	12	30	10	1	III A	4025.89...	40	5	.....	.....	IV E
3452.20d..	50	10	3	.....	III E	4031.70...	300	20	5	.....	III E
3453.16*..	70	8	2	.....	III E	4036.60...	8	.....	.....	.....	V E
3461.18...	25	50	20	2	III A	4037.22...	50	60r	50	50	I
3480.60...	8	30	10	1	III A	4042.92...	300	15	2	.....	IV E
3509.98...	25	4	.....	.....	IV E	4050.09...	60	.....	.....	.....	V E
3512.91...	25	3	.....	.....	IV E	4060.33...	60	40r	40	20	II
3514.06...	20	40	15	5	II A	4064.79...	50	30	30	15	II
3517.11*..	.....	.....	.....	.....	V E	4065.58...	30	25	25	12	II
3530.66†..	20	— ?	.....	.....	V (?) E	4067.40...	125	12	1	.....	IV E
3550.81†..	15	10?	.....	.....	IV (?) E	4076.70...	30	3	.....	.....	IV E
3557.25†..	20	5?	.....	.....	IV (?) E	4077.34d..	400	40	8	.....	III E
3574.41*..	50	?	20	10	II	4079.18...	40	40r	40	35	I
3593.29...	2	.....	.....	.....	V E	4086.72...	400	25	15	1	III E
3601.05...	15	1	.....	.....	V E	4089.62...	50	35	35	20	I
3612.35...	8	.....	.....	.....	V E	4099.55...	60	.....	.....	.....	V E
3613.10...	30	30r	10	5	II	4104.88...	60	30	30	25	I
3621.74...	4	.....	.....	.....	V E	4109.49...	15	30	25	5	II A
3628.83...	125	8	1	.....	IV E	4109.82...	20	40	30	20	I A
3636.67...	40	25	10	1	III	4117.69...	20	20	5	.....	III
3637.15d..	50	1	.....	.....	V E	4123.23...	600	25	10	.....	III E
3641.53*..	100	50r	15	8	II	4137.03...	40	40	40	35	I
3645.41...	400	15	2	.....	IV E						

TABLE IV—*Continued*

λ (I.A.)	Arc Int.	Furnace Intensities			Class	λ (I.A.)	Arc Int.	Furnace Intensities			Class
		High Temp.	Medium Temp.	Low Temp.				High Temp.	Medium Temp.	Low Temp.	
4141.72d..	80? 40?	8	.....	.....	IV E	4443.95d..	10	15	10	8	I
4143.95*..	12	25	15	3	III A	4445.12...	2	4	2	.....	III A
4151.95d..	300	20	4	.....	III E	4452.17...	30	20	15	5	II
4152.76†..	40	2?	.....	.....	IV (?) E	4453.85...	2	6	.....	.....	IV A
4157.49...	10	20	20	8	II A	4455.23...	10	20	15	8	II A
4160.25...	30	30	25	20	I	4455.82...	25	1	.....	.....	V E
4163.30...	8	25	20	2	III A	4468.97...	25	30	25	10	II
4171.10...	8	20	10	1	III A	4474.55...	5	10	3	.....	III A
4172.31...	10	20	15	2	III A	4479.82d..	15	30	30	10	II A
4177.47...	30	30	25	20	I	4486.07...	20	20	6	2	III
4187.31...	125	125R	100	100	I	4491.78d..	15	20	6	1	III
4192.35d..	80	.....	.....	.....	V E	4493.14...	25	30	25	20	I
4196.55...	300	15	6	.....	III E	4493.82...	10	20	10	8	I A
4204.04...	100	2	.....	.....	V E	4494.70...	30	25	20	15	I
4217.56d..	80	.....	.....	.....	V E	4499.07d..	10	15	5	.....	III
4230.96...	30	.....	.....	.....	V E	4500.24...	40	30	20	6	II
4238.40d..	400	30	4	.....	III E	4501.59...	10	20	8	4	II A
4238.61*..	10	20	15	3	III A	4522.38...	500	10	3	.....	III E
4250.01d..	25	.....	.....	.....	V E	4525.31d..	20	.....	.....	.....	V E
4256.93...	6	20	8	.....	III A	4526.12...	200	6	2	.....	III E
4262.35...	15	30	30	10	II A	4530.59...	2	.....	.....	.....	V E
4263.59d..	100	.....	.....	.....	V E	4541.78d..	15	15	8	2	III
4269.50d..	150	1	.....	.....	V E	4549.51...	50	20	20	20	I
4275.64...	60	8	1	.....	IV E	4550.16...	5	10	6	2	III A
4280.27...	100	100R	80	80	I	4550.78d..	10	20	6	1	III A
4286.98d..	150	tr	.....	.....	V E	4552.50...	8	15	5	2	II A
4296.05...	200	15	1	.....	IV E	4558.47...	250	12	3	.....	III E
4300.45...	30	6	tr	.....	IV E	4559.29...	50	1	.....	.....	V E
4306.00...	6	20	8	4	II A	4564.84...	12	20	15	4	III
4311.74d..	4	15	6	tr	III A	4567.92...	200	50	50	50	I
4315.91d..	12	tr	.....	.....	V E	4570.03...	250	60	60	60	I
4322.51...	150	15	5	.....	III E	4570.96...	8	.....	.....	.....	V E
4333.77...	600	30	20	1	III E	4574.89d..	300	10	4	.....	III E
4334.98...	60	2	.....	.....	V E	4580.07...	100	2	.....	.....	V E
4339.95...	6	20	30	4	III A	4581.20d..	12	12	6	1	III
4340.74...	15	15	15	tr	III	4596.17...	10	8	10	10	I
4354.40...	80	3	.....	.....	IV E	4602.03d..	20	15	8	1	III
4354.80...	25	30	20	2	III	4604.23...	10	6	2	.....	III
4357.92...	2	12	4	.....	III A	4605.09...	10	6	2	.....	III
4360.48...	2	8	2	.....	III A	4605.78...	50	.....	.....	.....	V E
4360.85...	2	6	2	.....	III A	4613.40*..	200	— ?	.....	.....	V E
4363.07...	4	.....	.....	.....	V E	4615.06d..	15	8	3	1	III
4364.68...	30	2	.....	.....	IV E	4619.88...	150	.....	.....	.....	V E
4378.11d..	15	1	.....	.....	IV E	4636.42...	4	.....	.....	.....	V E
4380.55...	12	20	30	6	II A	4643.12...	5	4	2	.....	III
4383.46...	25	.....	.....	.....	V E	4645.31...	40	.....	.....	.....	V E
4385.21...	20	.....	.....	.....	V E	4646.36...	12	5	2	.....	III
4389.89...	15	10	4	.....	III	4647.49d..	8	.....	.....	.....	V E
4393.61d..	4	3 3	2	.....	III	4648.66...	40	15	15	15	I
4396.31...	2	1	.....	.....	IV	4650.35...	15	6	6	4	I
4396.79d..	4	6	1	.....	IV A	4652.11†..	20	8?	5	4	I
4397.04...	2	2	.....	.....	IV A	4655.53d..	150	.....	.....	.....	V E
4402.66...	15	15	8	1	III	4660.73...	8	5	3	1	III
4403.03d?.	8	15	8	1	III A	4662.54...	200	6	2	.....	III E
4411.21...	2	.....	.....	.....	V E	4663.79d..	50	.....	.....	.....	V E
4413.44...	2	6	3	tr	III A	4668.92*..	50	.....	.....	.....	V E
4417.11...	6n	6	3	tr	III	4671.83*..	40	.....	.....	.....	V E
4419.16...	6	.....	.....	.....	V E	4688.66...	2	.....	.....	.....	V E
4423.91...	30	20	20	4	II	4691.16d..	30	.....	.....	.....	V E
4427.58d..	40	.....	.....	.....	V E	4692.50d..	50	.....	.....	.....	V E
4429.89d..	500	50	40	.....	III E	4699.62*..	20	.....	.....	.....	V E
4435.84...	6	2	.....	.....	IV E	4700.27†d.	8	6?	3?	.....	III?
4442.67...	12	15	15	4	II	4702.63...	10	8	8	5	I
						4703.27*..	20	.....	.....	.....	V E
						4708.17†d.	8	?	2?	.....	III?

TABLE IV—*Continued*

λ (I.A.)	Arc Int.	Furnace Intensities			Class	λ (I.A.)	Arc Int.	Furnace Intensities			Class
		High Temp.	Medium Temp.	Low Temp.				High Temp.	Medium Temp.	Low Temp.	
4712.90d..	12	.....	.....	.....	V E	4968.58...	2	.....	.....	.....	V
4714.13...	5	5	5	4	I			4	1	.....	III A
4716.41d..	25	.....	.....	.....	V E	4970.37...	100	4	1	.....	III E
4717.56...	4	.....	.....	.....	V E	4977.92...	8	20	20	5	II A
4719.90...	15	.....	.....	.....	V E	4984.91†..	2	1 ?	.....	.....	IV ?
4723.68...	3	5	4	2	II	4986.81d..	100	8	2	.....	III E
4724.39...	6	.....	.....	.....	V E	4991.26d..	20	1	.....	.....	IV E
4728.38†d.	80	?	.....	.....	V (?) E	4993.84...	20	25	30	15	II
4729.09...	1	.....	.....	.....	V	4994.64...	1	3	1	.....	III A
4733.80†d.	4	— ?	.....	.....	V ?	4996.79*..	2	.....	.....	.....	V E
4739.78...	2	.....	.....	.....	V E	4999.46*..	200	10	2	.....	III E
4740.25...	150	6	3	.....	III E	5001.78d..	10	20	10	.....	III A
4743.08d..	100	.....	.....	.....	V E	5002.10*..	3	.....	.....	.....	V E
4748.72...	100	1	.....	.....	V E	5019.51...	8	4	1	.....	III
4750.41...	15	8	4	1	III	5046.86d..	60	30	20	4	III
4752.38...	3	3	2	1	III	5048.00...	1	.....	.....	.....	V E
4753.13d..	1	tr	.....	.....	IV	5050.56...	80	20	20	10	II
4756.97...	1	.....	.....	.....	V	5056.45...	80	20	20	8	II
4757.16d..	2	1	.....	.....	IV	5062.89...	4	.....	.....	.....	V E
4759.71†d.	2	2	?	.....	IV?	5067.86d..	15	20	10	1	III
4766.91...	100	50	50	60	I	5072.12...	1	4	2	.....	III A
4770.45...	15	6	4	2	II	5078.94*..	2	?	.....	.....	V ?
4775.16d..	2	.....	.....	.....	V	5079.37...	5	2	tr	.....	IV
4779.90...	4	5	4	2	II	5080.20*..	1	.....	.....	.....	V E
4780.55d..	1	.....	.....	.....	V	5090.55*..	tr	.....	.....	.....	V E
4791.39...	5	6	8	4	II	5096.62†..	1	?	?	.....	IV ?
4792.46...	tr	2	1	.....	III A	5103.13...	2	2	1	.....	III
4794.57*..	2	.....	.....	.....	V E	5106.23†d.	150	100 ?	100	50	II
4796.68...	4	.....	.....	.....	V E	5109.11†..	2	5 ?	3	.....	III A
4800.00...	8	4	3	tr	III	5114.55†d.	150	?	?	.....	III (?) E
4800.25...	8	6	5	1	III	5120.86...	10	5	3	.....	III
4804.04...	50	2	.....	.....	V E	5122.98...	150	3	1	.....	III E
4809.00...	80	2	.....	.....	V E	5135.45...	2	.....	.....	.....	V
4817.15*..	4	4	?	.....	IV ?	5145.42...	200	125	125	60	II
4824.04...	80	2	1	.....	III E	5156.74...	30	.....	.....	.....	V E
4826.85...	4	.....	.....	.....	V E	5157.44*..	15	.....	.....	.....	V E
4830.50...	1	.....	.....	.....	V E	5158.69...	80	60 ?	60	50	I
4839.51...	25	10	10	6	II	5163.62†..	20	?	.....	.....	V (?) E
4840.00...	10	.....	.....	.....	V E	5164.03†..	1	—?	.....	.....	V ?
4843.27...	1	.....	.....	.....	V	5167.78...	20	10	10	2	III
4850.56...	5	.....	.....	.....	V E	5172.90*..	1	.....	.....	.....	V E
4850.80...	20	20	20	30	I	5173.85*..	10	?	?	.....	IV (?) E
4854.93...	8	6	6	1	III	5177.30d..	300	200	200	100	II
4860.88...	100	6	3	.....	III E	5179.11...	2	2	1	.....	III
4868.90†..	2	2 ?	.....	.....	IV ?	5183.41*..	400	5	2	.....	III E
4870.55...	5	5	4	1	III	5183.89...	20	20	20	10	II
4878.84...	15	10	10	2	III	5188.20*..	25	.....	.....	.....	V E
4886.81...	2	2 ?	.....	.....	IV	5204.15*..	12	.....	.....	.....	V E
4887.60...	5	8	4	tr	III	5211.87d..	300	250	250	60	II
4894.23...	1	1	tr	.....	III	5226.19*..	1	.....	.....	.....	V E
4899.91...	300	20	10	.....	III E	5234.28d..	300	250	250	50	II
4901.86...	25	30	20	20	I	5239.55...	4	4	3	.....	III
4905.11...	4	8	4	tr	III A	5240.81...	2	tr	tr	.....	III
4916.60†..	1	2	?	.....	III (?) A	5253.45...	100	50	50	50	I
4920.95...	400	15	12	.....	III E	5257.85...	3	1	tr	.....	III
4921.77...	400	15	10	.....	III E	5259.38...	40	1	tr	.....	III E
4925.37...	2	2	.....	.....	IV	5271.19...	150	100	100	80	I
4934.82...	20	1	.....	.....	V E	5276.42...	10	6	5	.....	III
4935.59...	2	.....	.....	.....	V E	5279.13...	2	.....	.....	.....	V E
4945.82...	5	12	7	.....	III A	5287.43...	1	.....	.....	.....	V
4946.42*d.	20	?	?	.....	IV (?) E	5290.84...	60	2	1	.....	III E
4949.76...	200	100r	100	100	I	5301.98d..	200	10	10	tr	III E
4952.03*..	5	.....	.....	.....	V E	5302.63*..	40	.....	.....	.....	V E
4956.05...	1	.....	.....	.....	V	5303.55...	100	1	1	.....	III E
4964.80...	4	15	8	.....	III A	5304.01...	30	10	10	tr	III

TABLE IV—*Continued*

λ (I.A.)	Arc Int.	Furnace Intensities			Class	λ (I.A.)	Arc Int.	Furnace Intensities			Class
		High Temp.	Medium Temp.	Low Temp.				High Temp.	Medium Temp.	Low Temp.	
5307.54...	3	1	tr	.....	III	5712.41d..	20	1	1	.....	III E?
5320.16...	3	5	5	tr	III	5714.02...	5	10	10	1	III A
5323.57...	3	2	2	.....	III	5714.55d..	1	2	1	.....	III A
5340.69*..	40	8	8	tr	III E	5720.03d?.	10	40	30	4	III A
5357.87...	60	15	15	2	III	5727.30...	4	.....	.....	.....	V E
5359.72...	2	5	4	.....	III A	5734.95...	5	10	7	.....	III A
5365.89...	8	6	5	.....	III	5740.65...	100	80	80	80	I
5377.09*..	2	.....	.....	.....	V E	5742.95d..	2	8	6	tr	III A
5380.01...	5	3	3	.....	III	5744.42d..	80	60	40	4	III
5380.99...	80	tr	.....	.....	V E	5761.84...	60	60	60	50	I
5381.90*..	15	.....	.....	.....	V E	5769.07...	50	.....	.....	.....	V E
5415.66...	8	6	4	.....	III	5769.37...	80	80	70	60	I
5422.10...	3	6	4	tr	III A	5769.99...	25	70	40	8	III A
5429.91...	15	10	8	1	III	5789.23d?.	250	200	200	150	I
5437.54...	5	4	3	.....	III	5791.33d?.	400	400	400	250	I
5455.13...	400	200	200	150	I	5797.59...	50	10	3	.....	III E
5458.67d..	20	.....	.....	.....	V E	5802.08d..	1	4	2	.....	III A
5464.36...	25	.....	.....	.....	V E	5805.77...	50	10	3	.....	III E
5466.90...	3	5	3	.....	III	5808.29*..	15	2?	tr?	.....	IV?
5475.15d..	15	10	6	tr	III	5813.39...	1	3	1	.....	III A
5480.71...	1	.....	.....	.....	V E	5822.00...	30	50	30	3	III
5482.26...	50	.....	.....	.....	V E	5823.83...	10	30	15	2	III A
5486.80...	3	.....	.....	.....	V	5827.55d..	3	10	6	1	III A
5491.04...	8	8	6	1	III	5829.73d..	10	50	30	4	III A
5493.44...	20	.....	.....	.....	V E	5839.79...	2	20	12	5	II A
5498.70...	2n	1	tr	.....	III	5845.02...	6	50	50	20	II A
5501.35...	300	150	150	125	I	5848.36d..	8	15	6	1	III A
5502.23...	4	2	2	.....	III	5852.27...	2	15	6	1	III A
5502.66...	10	10	8	2	III	5855.58...	15	40	40	20	II A
5503.86...	80	40	50	6	III	5863.68...	20	tr	.....	.....	V E
5506.07...	40	50	75	20	II	5869.97...	1	6	3	.....	III A
5507.33...	2	1	1	.....	III	5873.99...	3	tr	.....	.....	IV
5515.32d..	10	8	8	tr	III	5874.73...	8	15	12	3	III A
5517.39d..	30	8	8	tr	III	5877.63...	2	15	12	4	III A
5524.40...	1	.....	.....	.....	V	5877.98...	1	4	3	.....	III A
5526.51...	2	.....	.....	.....	V	5880.62...	30	2	1	.....	III E?
5529.90...	3	2	2	.....	III	5892.66...	1	.....	.....	.....	V
5532.07...	3	4	4	1	III	5894.83d..	8	20	15	3	III A
5535.67...	15	.....	.....	.....	V E	5900.72d..	1	4	2	.....	III A
5541.28d..	20	8	8	1	III	5901.96...	2	tr	.....	.....	IV E
5544.92...	6	2	2	.....	III	5904.30...	2	10	6	.....	III A
5562.54†..	2	1	1?	.....	III?	5917.62...	15	40	40	25	II A
5565.40d..	20	10	10	1	III	5928.50...	4	12	6	1	III A
5565.74...	20	15	15	5	II	5930.60...	400	400	400	300	I
5566.94*..	2	.....	.....	.....	V E	5935.26...	15	40	40	20	II A
5568.48...	50	30	40	12	II	5936.22...	12	1	.....	.....	V E
5570.39...	5	12	12	4	II A	5940.84d..	1	4	3	.....	III A
5588.35...	80	40	60	20	II	5960.58...	3	8	5	tr	III A
5631.22...	100	20	20	20	I	5962.58...	1	4	2	.....	III A
5632.06...	25	10	5	2	II	5965.33...	1	1	1	.....	III
5639.30d?.	5	10	6	1	III A	5973.51...	5	.....	.....	.....	V E
5648.26d..	80	80	20	2	III	5975.79d?.	3	12	7	1	III A
5654.84d..	3	15	4	1	III A	5982.34...	5	20	20	6	III A
5656.57...	1	12	2	.....	III A	5992.38d..	2	8	5	.....	III A
5657.74d?.	50	60	80	25	II	6007.37...	50	200	150	15	III A
5661.37d..	1	5	2	.....	III A	6017.22...	1	4	2	.....	III A
5669.97...	1	1	.....	.....	IV	6025.11†..	1	?	.....	.....	IV?
5671.56...	8	.....	.....	.....	V E	6031.49†..	3n	?	.....	.....	IV?
5696.19...	40	40	40	40	I	6034.55...	3	.....	.....	.....	V
5699.38...	5	6	6	1	III	6035.12...	2	.....	.....	.....	V
5700.25...	3	3	3	.....	III	6038.60...	25	50	20	4	III A
5701.15...	1	1	1	.....	III	6041.59...	2	2	1	.....	III
5702.57...	2	2	2	.....	III	6044.85...	2	4	1	.....	III A
5703.29...	10	15	15	3	III	6046.05†..	1	1?	.....	.....	IV?
5710.81...	2	1	1	.....	III	6061.43†..	2	—?	.....	.....	V?

TABLE IV—*Continued*

λ (I.A.)	Arc Int.	Furnace Intensities			Class
		High Temp.	Medium Temp.	Low Temp.	
6067.15†..	1	— ?	.....	.....	V ?
6068.70d..	30	50	20	4	III
6072.06...	4	10	3	1	III A
6074.01...	1	4	1	.....	III A
6075.24...	1	4	1	.....	III A
6084.93...	8	6	1	.....	III
6085.46...	1n	.....	.....	.....	V
6087.95...	1n	1	.....	.....	IV
6092.27...	1	3	1	.....	III A
6100.36...	10	.....	.....	.....	V
6107.27...	12	60	40	10	II A
6108.50...	60	80	40	8	II
6111.73...	30	50	15	5	II
6120.34...	1	1	.....	.....	IV
6121.27...	1	1	.....	.....	IV
6123.81...	2	1	.....	.....	IV
6126.08†..	10	— ?	.....	.....	V ?
6127.05...	12	40	10	2	III A
6129.55†..	15	1 ?	.....	.....	IV ?
6134.40d..	30	50	25	6	III
6136.53...	2	6	2	1	II A
6143.99d..	10	12	2	.....	III
6145.34...	3	4	1	.....	III
6146.55...	5	1	.....	.....	IV
6165.04...	2	3	tr	.....	IV
6165.72...	40	60	40	10	II
6172.73†..	5	— ?	.....	.....	V
6173.74*..	1	3	tr	.....	IV A
6174.19...	1	.....	.....	.....	V
6188.11†..	1	— ?	.....	.....	V
6203.54†..	2	— ?	.....	.....	V
6218.22...	7	10	2	.....	III
6219.52...	2	.....	.....	.....	V
6233.52...	15	50	12	2	III A
6234.84d..	15	40	10	2	III A
6236.23...	12	6	1	.....	IV
6236.78d..	10	25	8	2	III A
6238.59...	15	60	15	2	III A
6249.92...	500	300	500	400	I
6262.30*..	50 ?	8 ?	2	.....	III
	30 ?	4 ?			
6266.03d..	60	100	20	2	III
6273.78*..	2n	2 ?	1 ?	.....	III ?
6287.75d..	8	15	3	tr	III A
6288.58...	7	15	3	.....	III A
6293.57...	80	200	200	50	II A
6296.10...	50	3	.....	.....	IV E
6305.46...	5	5	1	.....	IV
6307.27...	tr	tr	.....	.....	IV
6308.27...	3	4	1	.....	III
6308.99...	2	3	.....	.....	III
6310.22...	12	10	.....	.....	IV
6310.94...	4	.....	.....	.....	V E
6315.82...	1	.....	.....	.....	V
6318.29...	12	40	10	2	III A
6320.39...	80	20	3	.....	III
6325.92...	150	100	250	200	I
6330.48...	2	10	3	tr	III A
6333.22†..	2	1 ?	1 ?	.....	III ?
6333.83†..	2	2 ?	.....	.....	IV ?
6337.92...	1	.....	.....	.....	V
6339.25†..	2	10 ?	3	1	III A
6356.44...	4	8	2	.....	III A
6358.13...	5	1	.....	.....	IV
6360.24...	30	30	10	4	II
6374.14...	1	.....	.....	.....	V
6375.11...	2	.....	.....	.....	V
6380.56...	1	.....	.....	.....	V
6390.45...	80	20	4	.....	III
6394.24...	600	400	500	500	I
6299.06†..	10	— ?	— ?	.....	V (?) E
6410.98...	300	200	250	200	I
6417.22†..	3	10 ?	2 ?	1	III (?) A
6443.09...	1	.....	.....	.....	V
6446.62...	5	.....	.....	.....	V E
6448.15*..	50 ?	100 ?	75 ?	30 ?	II A
	10 ?	30 ?	25 ?	10 ?	II A
6450.33...	8	20	4	2	II A
6454.51d..	200	200	200	200	I
6455.98...	300	200	250	250	I
6468.44d..	8	20	5	2	II A
6485.54d..	20	40	10	5	II A
6492.90...	5	.....	.....	.....	V
6498.19†..	7	1 ?	.....	.....	IV (?) E
6506.27...	6	6	1	.....	IV
6519.33...	4	3	.....	.....	IV
6520.77...	20	6	1	.....	IV
6523.90†..	2	10 ?	3 ?	1 ?	III A ?
6526.98d..	100	20	2	1	III E
6529.76†d.	4	?	?	.....	III ?
6543.15d..	500	400	500	500	I
6549.19†..	2n	5 ?	1 ?	.....	III ?
6554.14...	1	.....	.....	.....	V
6555.05...	1	.....	.....	.....	V
6555.95...	3	10	3	1	III A
6565.42...	40	80	80	80	I A
6570.94...	2n	2	1	.....	III
6578.52...	400	300	400	400	I
6582.21...	6	6	1	.....	IV
6593.46...	60	80	80	60	I
6599.48...	tr	1	.....	.....	IV A
6600.17...	50	100	100	50	II A
6607.68...	3	1	.....	.....	IV
6608.25...	40	40	12	5	II
6616.59...	80	125	125	100	I
6628.39...	1	2	.....	.....	IV A
6631.22†..	3	— ?	.....	.....	V ?
6636.52...	1	.....	.....	.....	V
6642.78...	3	.....	.....	.....	V E
6644.39d?.	40	125	150	125	I A
6645.19...	8	8	tr	.....	IV
6650.80...	100	200	300	300	I A
6661.41d?.	80	150	150	125	I A
6671.40fd.	6	1 ?	.....	.....	IV (?) E
6692.88...	30	60	60	40	I A
6699.24†..	2	1 ?	1 ?	.....	III ?
6699.84...	4	8	2	tr	III A
6709.50...	200	150	200	200	I
6714.09...	2	.....	.....	.....	V E
6716.01†..	1	3 ?	.....	.....	IV A ?
6718.66...	tr	.....	.....	.....	V E
6732.79...	1	.....	.....	.....	V E
6748.13...	10	20	5	3	II A
6753.04...	50	100	150	150	I A
6774.27d?.	30	12	2	1	III E
6776.79...	1	6	1	.....	III A
6778.24...	tr	.....	.....	.....	V
6783.50...	tr	1	.....	.....	IV A
6796.71...	4	40	15	12	II A
6808.85d..	6	3	.....	.....	IV

TABLE IV—*Continued*

λ (I.A.)	Arc Int.	Furnace Intensities			Class	λ (I.A.)	Arc Int.	Furnace Intensities			Class
		High Temp.	Medium Temp.	Low Temp.				High Temp.	Medium Temp.	Low Temp.	
6813.66†..	1	— ?	.....	.....	V (?) E	7149.70...	2	2	2	.....	III
6816.29†..	1n	— ?	.....	.....	V ?	7158.06d..	50	100	100	30	II A
6821.53...	3n	.....	.....	.....	V	7161.19...	50	50	25	5	III
6823.79...	15	30	12	10	II A	7219.88...	15	30	15	6	II A
6830.82*..	tr	.....	.....	.....	V E	7270.08...	10	5	3	.....	III
6834.06†..	4	3 ?	.....	.....	IV (?) E		5	15	10	6	II A
6837.91†..	3	2 ?	.....	.....	IV (?) E	7282.32*..	100	10	2	.....	III E
6859.04...	tr	.....	.....	.....	V	7334.15d..	60	100	100	50	II
6898.41...	tr	.....	.....	.....	V	7345.31d..	20	40	10	1	III A
6899.55...	tr	.....	.....	.....	V	7379.71...	10	20	3	.....	III A
6902.12...	1	1	tr	.....	III	7382.67...	5	10	2	.....	III A
6903.07...	1	4	1	.....	III A	7463.08d..	10	15	4	.....	III
6917.24...	10	20	10	2	III A	7483.45*...	20	5	.....	.....	IV E
6918.27...	8	40	20	4	III A	7498.76...	5	10	2	.....	III A
6925.24...	100	30	15	4	III	7539.20...	15	20	20	10	II
6934.98...	50	60	30	8	III	7664.33*..	?	15	5	.....	III
6952.50d?*	2	2	.....	.....	IV	7841.86...	2	8	4	.....	III A
6954.51d..	8	4	tr	.....	IV	7947.92d..	10	15	10	.....	III
6958.09...	20	3	.....	.....	IV E	7964.83...	6	10	8	.....	III
6968.72...	tr	.....	.....	.....	V E	8001.96...	4	10	8	.....	III A
6976.84...	3	2	tr	.....	III	8051.36d..	10	15	5	.....	III
7023.63d?.	150	100	25	2	III	8086.07...	15	15	10	.....	III
7032.01...	15	10	5	.....	III	8247.48...	10	10	5	.....	III
7045.93d?.	300	200	200	80	II	8316.03...	2	5	.....	.....	IV A
7066.19d?.	400	10	2	.....	III E	8324.69...	15	10	5	.....	III
7068.32...	100	80	80	40	II	8346.55...	12	8	5	.....	III
7076.37...	3	3	3	tr	III						

NOTES—LANTHANUM

3010.80 Low-temperature spectrum fades out, and class may be too high.
3171.66 Strongest spark line in this region.
3193.00 Doublet. Not fully resolved.
3453.16 Probably triple.
3517.11 Very strong spark line. Cf. λ 3171.66.
3574.41 Reversed at high temperature. Obscured by cyanogen band
3641.53 Enhanced line on red side.
3895.65 Blend *Fe*.
3010.80 Furnace line may be impurity.
4143.95 May be impurity.
4238.61 Measured by writers.
4613.40 Faint if present in furnace.
4668.92 Probably triple.
4671.83 Triple, violet member just resolved
4699.62 Probably triple.
4703.27 Probably triple.
4794.57 Widens to violet in spark.
4817.15 Probably triple.
4946.42 Blend impurity line.
4952.03, 4996.79 } Much enhanced
4999.46 Triple.
5002.10 Much enhanced.
5078.94 Furnace line may be impurity.
5080.20, 5090.55, 5157.44, 5172.90 } Much enhanced.
5173.85 Blend *Ti* in furnace.
5183.41 Triple.
5188.20, 5204.15, 5226.19, 5302.63 } Much enhanced.
5340.69 Much enhanced. Probably coincides with arc line.

TABLE IV—*Continued*

5377.09 5381.90 5566.94	Much enhanced.
5808.29	Blend with line to violet.
6173.74	Measured by writers.
6262.30	Not fully resolved.
6273.78	Furnace line may be impurity.
6448.15	Not fully resolved.
6830.82	Measured by writers.
6952.50	May be triple.
7282.32	Triple. Red component strongest.
7483.45	Triple.
7664.33	Blend potassium l'ne in arc.

obtained whose difference was large enough to be significant. In order to retain a consistent classification, the lack of a medium temperature was allowed for as far as possible. This resulted in placing in class III those lines which are strong at high temperature but much weaker at low temperature, while lines which retain considerable strength at low temperature are assigned to class II or class I.

Symbols.—The significance of the symbols and abbreviations found in the tables is as follows: * refers to a note at the end of the table; † indicates disturbance by a band spectrum, usually that of carbon, which causes some of the intensities to be questioned; "d," after the wave-length, indicates an unresolved doublet; "tr," a line showing as trace, less than intensity 1; "r," "R," partial or complete self-reversal; "n," "N," degrees of diffuseness in arc lines; "A," after the class number, a line relatively stronger in the furnace than in the arc; "E," after the class number, an enhanced line.

DISCUSSION

A summary of the types of the 2005 lines classified in this paper is given in Table V. Multiplets corresponding to low energy-levels will normally be found in the low-temperature groups, but may include a few lines which are placed in higher temperature classes on account of faintness, which prevents their appearing on low-temperature spectrograms of ordinary exposure, or which are uncertain on account of blends. The lines which appear with considerable strength at low temperature include from 15 to 30 per cent of the neutral-atom lines listed for these elements.

A large proportion of the lines which appear in the arc spectra of these three elements are enhanced lines, those having sufficient strength to be listed in the tables numbering 85, 342, and 230 for

yttrium, zirconium, and lanthanum, respectively. Of these, more than half appear in the furnace for yttrium and lanthanum, but only a small proportion (all of wave-length shorter than λ3450) in the case of zirconium. The lists are by no means complete for the enhanced lines, as many additional ones are given by a strong spark. We have, however, segregated the enhanced lines by this means into two groups requiring different stages of excitation, which correspond in some degree with the low- and high-temperature lines of the neutral atom. Enhanced lines which appear in the furnace, especially those distinct at medium temperature, may be expected to arise from low energy-levels of the ionized atom, while those requiring the arc excitation should belong to higher levels. A third group of

TABLE V

DISTRIBUTION OF ARC AND ENHANCED LINES ACCORDING TO LOW AND HIGH EXCITATION

	Y	*Zr*	*La*
Low-temperature lines (classes I and II)	69	142	145
High-temperature arc lines (in some cases uncertain through faintness)	305	375	320
Enhanced lines appearing in furnace	47	22	128
Enhanced lines in arc but not in furnace	38	320	102

still more refractory enhanced lines, possibly in some cases belonging to the second ionization, would contain those lines which are absent from the arc but appear in a highly condensed spark. For both yttrium and lanthanum, many strong enhanced lines in the short wave-length region persist in the furnace at 2100°C.

The very frequent occurrence of doublets, among both arc and enhanced lines, is a notable feature of the lanthanum spectrum. Those thus designated in Table IV number 127, or 18 per cent of the lanthanum lines listed, and the actual number is probably greater, as the first-order dispersion used beyond λ 5000 was not always sufficient to show the doublet character. The wave-length tables available give these lines as single. The separation of the components is probably always less than 0.08 A, as in only a few cases were they resolved enough in the vacuum furnace to give a rough estimate of the separate intensities. Some of these imperfectly resolved lines are

clearly triple and are so noted in the remarks at the end of Table IV; it is probable that other triplets occur among the lines marked as double.

Lines relatively stronger in the furnace than in the arc, indicated by "A" after the class number, are in about the usual proportion in the yttrium list, while the high temperatures required for zirconium make the spectrum difficult for the furnace and only one line is thus designated. For lanthanum the proportion is unusually large, 127 such lines being found. While the number of these lines is regulated by the average strength of lines in the arc spectrograms selected for comparison with those of the furnace, it is significant in the case of lanthanum, because a large proportion of lines strong in the arc are enhanced lines, while the furnace is highly effective in giving the lines of the neutral atom.

MOUNT WILSON OBSERVATORY AND VASSAR COLLEGE
September 1926

THE ABSOLUTE MAGNITUDES OF GIANT M STARS[1]

By GUSTAF STRÖMBERG

ABSTRACT

Derivation of absolute magnitudes from spectroscopic criteria.—The article gives the method and the formulae used in deriving the reduction curves which are the basis of the absolute magnitudes of *giant M stars* published by Adams, Joy, and Humason in *Mt. Wilson Contr.*, No. 319. Provisional curves for each spectral subdivision were formed for each line listed in *Contr.* No. 319 and adjusted to give an accordant system of magnitudes M_1. Systematic corrections, S, to this provisional system then gave the adopted system M_1+S. The absolute magnitudes were derived entirely from the *peculiar motions;* parallactic motions have not been used because of the large dispersion in velocity. Results from trigonometric parallaxes are also given, but the parallaxes are very small and not numerous enough to give the same accuracy that can be obtained from the peculiar motions.

Probable errors and residual systematic corrections.—Figure 1 shows the relation between the computed absolute magnitudes and the provisional magnitudes. The deviations from the line represent the *residual systematic corrections*, S' (Table I), to the adopted magnitudes. In general these are below 0.1 mag. S_1' and S_2' are similar corrections for use in deriving mean parallaxes and mean tangential motions. The *probable errors* of individual determinations of an absolute magnitude for the supergiants and the ordinary giants are ± 0.63 and ± 0.45 mag., respectively. These were derived from the dispersion in M, which was obtained from a simple formula, equation (10), which is not influenced by the systematic corrections. This formula is also applicable in cases where no spectroscopic absolute magnitudes are available, in which case τ' is a reduced τ-component.

In *Mount Wilson Contribution*, No. 319,[2] Adams, Joy, and Humason have published the absolute magnitudes, as determined from spectroscopic criteria, of practically all the M stars brighter than apparent magnitude 6.0 and north of declination $-30°$. All of these stars, at least as far as the spectroscopic criteria indicate, are giants. The reduction curves used for the different spectral lines were derived by the writer. The following account gives data regarding the systematic and accidental errors affecting the results.

The general method used by Adams was to select lines which apparently varied with absolute magnitude. As an indication of the general order of absolute magnitude, the reduced proper motion $H=m+5 \log \mu$ was used. The existence of a correlation between line-intensity and H was accepted as an indication that a line could

[1] *Contributions from the Mount Wilson Observatory*, No. 327.

[2] *Astrophysical Journal*, **64**, 225, 1926.

be used for determining the absolute magnitude. The lines finally used are given in the paper cited.

For each line groups of stars were formed having a small range in line-intensity and spectral subdivision. An approximate mean absolute magnitude was then computed from the peculiar motions by methods explained later on. The resulting reduction curves for the different lines were slightly adjusted with the aid of the correlation data for individual lines, so that all the lines would give, as nearly as possible, the same absolute magnitude. The absolute magnitudes derived from the adjusted curves are denoted by M_1. Corrections to this system were then computed with the aid of additional data which had been obtained in the meantime. The results of this last computation are given below, together with the derivation of the formulae for computing the mean absolute magnitudes and their dispersion.

The following notation is used:

M = the true absolute magnitude of a star; M_1 = the provisional absolute magnitude derived from spectroscopic criteria; S = the systematic correction to a mean M_1 to reduce it to a mean M; Δ = the difference between M and M_1+S for an individual star; m = the apparent visual magnitude from the *Henry Draper Catalogue;* π = the true parallax; π_1 = the spectroscopic parallax derived from the difference $M_1 - M$; τ = a peculiar angular cross-motion, either perpendicular to the great circle toward the assumed solar apex (τ-component), or a proper-motion component along any axis after the parallactic motion projected on this axis, calculated with π_1 as an approximate value of the parallax, has been subtracted; τ' = an angular peculiar motion divided by the parallax π_1 for the same star; τ_m and τ'_m = the average values of τ and τ' taken without regard to sign; θ = average peculiar radial velocity in km/sec., without regard to sign; ϵ = the peculiar linear cross-motion projected on the same axis as τ; $k = 4.738$ km × year/sec., numerically equal to the mean distance of earth to sun in kilometers divided by the number of seconds in a year.

We have then

$$M = M_1 + S + \Delta \,, \tag{1}$$

$$\pi = 0.1 \times 10^{0.2(M-m)} = \pi_1 \sigma \delta \,, \tag{2}$$

where

$$\left.\begin{aligned} \pi_1 &= 0.1 \times 10^{0.2(M_1-m)} \\ \sigma &= 10^{0.2S} \,; \qquad \delta = 10^{0.2\Delta} \end{aligned}\right\} \tag{3}$$

$$k\tau = \pi\epsilon = \pi_1 \sigma \delta \epsilon \,. \tag{4}$$

The errors Δ are supposed to be distributed according to a normal error function around a zero value, and the dispersion is defined by the equation

$$q^2=\overline{\Delta^2}\,.$$

We have

$$\bar{M}=\bar{M}_1+S\,,$$

where a bar above a symbol denotes an algebraic mean.

Further,

$$\bar{\delta}=\overline{10^{0.2\Delta}}=\frac{1}{\sqrt{2\pi}\,q}\int_{-\infty}^{+\infty}10^{0.2\Delta}\,e^{-\frac{\Delta^2}{2q^2}}\,d\Delta=A\,. \tag{5}$$

The following relation between A and q is then found:

$$q^2=50\ \mathrm{Mod}\log A=21.71\log A\,. \tag{6}$$

The approximate formula

$$A=1+\frac{0.02q^2}{\mathrm{Mod}^2}$$

was given in *Contribution* No. 199, page 6,[1] and can be used for q less than about ± 1.0, but the general formula (6) is equally simple.

Before forming means of the equations (4), we must consider the correlations between δ and the other quantities present. The errors Δ arise from several causes. First, there are errors in the measurements of the line intensities; further, the true line intensity may vary, the true absolute magnitude remaining constant; and, finally, the true absolute magnitude may vary while the spectral criteria used remain unaffected. We may thus expect correlations between Δ and M_1 and between Δ and M, the former being particularly marked if the true absolute magnitude varies less than that spectroscopically determined, and the latter, if the reverse is the case. In equation (4) we may thus expect a correlation between δ and π_1, and also between δ and π, and, consequently, between δ and τ. The correlation between absolute magnitude and velocity would produce a correlation between δ and ϵ, but, since the change in mean peculiar radial velocity for the different groups is in this case practically zero, we need not consider the last correlation.

[1] *Astrophysical Journal*, **53**, 18, 1921.

Forming the mean for equations (4), disregarding their signs, we find

$$k\tau_m = \bar{\pi}\theta = \overline{\pi_1\delta}\,\sigma\theta\ .$$

We thus obtain

$$\bar{\pi} = \frac{k\tau_m}{\theta} = \overline{\pi_1\delta}\,\sigma = \sigma_1\bar{\pi}_1\ . \tag{7}$$

We cannot here separate $\overline{\pi_1}$ from $\bar{\delta}$.

Dividing equation (4) by π_1 and forming similar means, we find

$$\sigma\bar{\delta}\theta = k\tau'_m\ .$$

Since, by (5), $\bar{\delta}$ is denoted by A, we may write

$$\sigma A = \frac{k\tau'_m}{\theta} = \sigma_2\ . \tag{8}$$

To determine σ and A separately, we must have recourse to moments of second order. First, we derive from a relation analogous to (5),

$$\bar{\delta^2} = \overline{10^{0.4\Delta}} = A^4 = \bar{\delta}^4\ . \tag{9}$$

Dividing equations (4) by π_1, and forming the means of the squares, we find

$$k^2\overline{\tau'^2} = \sigma^2 A^4\bar{\epsilon}^2\ .$$

From this and equation (8) we obtain

$$A^2 = \frac{\overline{\tau'^2}\,\theta^2}{\tau'^2_m\,\bar{\epsilon}^2}\ . \tag{10}$$

Here for $\bar{\epsilon}^2$ we may use the mean-square peculiar radial velocity.

When the proper motions themselves are small, the values of τ_m and τ'_m in equations (7) and (8) are systematically affected by the errors in the measurements. To correct them, I have used the formulae

$$\frac{\text{Corr. }\tau_m}{\tau_m} = \frac{\text{Corr. }\tau'_m}{\tau'_m} = \sqrt{1 - \frac{\eta^2}{\tau'^2_m}}\ ,$$

where η is the average error, assumed to be $\pm 0''.003$, in a component of the proper motion.

To increase the accuracy, the proper motions have been corrected for the systematic errors in right ascension as given in Boss's *Catalogue*, page xxviii. To the proper motions in declination, Raymond's corrections[1] were applied. Further, use was made not only of the τ-components, but also of the components of proper motion in right ascension and declination, corrected for parallactic motion with the aid of π_1 as an approximate value of the parallax. The formulae used are

$$\tau_\alpha = \mu_1 - B_1\pi_1 \; ; \qquad \tau_\delta = \mu_2 - B_2\pi_1$$
$$kB_1 = x_0 \sin\alpha - y_0 \cos\alpha$$
$$kB_2 = x_0 \cos\alpha \sin\delta + y_0 \sin\alpha \sin\delta - z_0 \cos\delta \, ,$$

where x_0, y_0, and z_0 are the equatorial components of the sun's motion. This motion has been assumed throughout to have the co-ordinates $\alpha = 270°$, $\delta = +30°$, $v = 20$ km/sec. Although the solar motion as derived from the radial velocities of the M stars has a value of about 23 km/sec., this difference has no appreciable effect upon the mean peculiar motions. In the computation of A^2, only the actual τ-components were used. As long as S and σ are small, there is no systematic difference between the results from the τ-components and from τ_α and τ_δ.

The quantity A and the corresponding dispersion can be computed from equation (10) only for fairly large groups. If the dispersion thus found is assumed to hold also for the smaller groups, individual determination of σ can be made for these groups.

The three quantities σ, σ_1, and σ_2 represent the factors by which the provisional spectroscopic parallaxes π_1 must be multiplied in using them to obtain the most probable absolute magnitudes, the mean parallaxes, and the mean linear tangential motions. The corresponding corrections to M_1 are S, S_1, and S_2, defined by the equations

$$S = 5 \log \sigma \, , \qquad S_1 = 5 \log \sigma_1 \, , \qquad S_2 = 5 \log \sigma_2 \, .$$

The data computed by these formulae are collected in Table I. The third and the last row may be regarded as representative for

[1] *Astronomical Journal*, **36**, 129, 1926.

TABLE I

Sp.	M_1	No.	π_1	θ km/sec.	S	S_1	S_2	r	$\overline{M}_1$	$\overline{M}_1+S$	S'	S'_1	S'_2
Mo–M2...	≤ -1.0	24	0".00428	16.17	+0.34	+0.25	+0.54	±0.63	−2.16	−1.82	−0.16	−0.25	+0.04
M3–M5...	≤ -1.0	20	.00558	17.60	.69	.61	.90	.63	−1.25	−0.56	+ .19	+ .11	+ .40
Mo–M5...	≤ -1.10	44	.00487	16.82	.52	.42	.72	.63	−1.75	−1.23	+ .02	− .08	+ .22
Mo–M2...	−0.9–−0.5	49	.00620	17.38	.36	.27	.46	.45	−0.64	−0.28	− .02	− .10	+ .09
Mo–M2...	≥ -0.4	93	.00749	17.93	.32	.34	.42	.45	− .15	+ .17	+ .01	+ .02	+ .11
Mo–M2...	$\geq +0.1$	19	.00899	16.2	.14	.27	.25	.45	+ .22	+ .36	− .13	− .01	− .03
M3–M5...	≥ -0.9	70	.00673	17.35	.27	.34	.37	.45	− .48	− .21	− .09	− .01	+ .01
Mo–M7...	≥ -0.9	223	0.00686	17.86	+0.27	+0.30	+0.37	±0.45	−0.39	−0.12	−0.08	−0.05	+0.03

the supergiants and the ordinary giants, respectively, although no sharp dividing limit can be given. The quantity r is the probable error in the spectroscopic absolute magnitude for a single star, and is simply the dispersion q converted into probable error. An important feature is the small variation in θ, which is derived from the radial velocities for the same stars whose proper motions are used.

Figure 1 shows the relationship between $\overline{M}_1$ and the computed mean absolute magnitude $\overline{M}$, which is equal to $\overline{M}_1+S$. The line

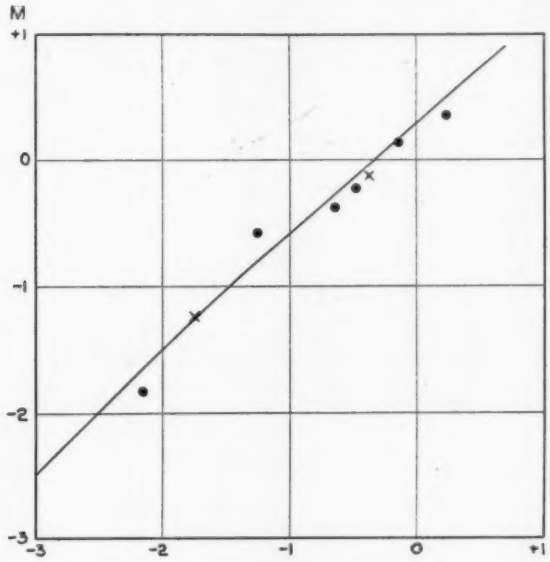


FIG. 1.—Correlation between absolute magnitudes M_1, derived from line intensities by using a provisional system of reduction curves, and the absolute magnitudes M derived from peculiar motions.

drawn was used in deriving the systematic corrections to the system, and the residuals are denoted by S', S'_1, and S'_2. These quantities may be regarded as the residual systematic corrections to the absolute magnitudes, as published by Adams, Joy, and Humason, which are to be applied according as it is desired to find absolute magnitudes, mean parallaxes, or mean tangential motions. They are all smaller than their mean errors and can well be neglected.

Appreciably different reduction curves have been used for the different spectral subdivisions. In *Contribution* No. 199, on the other

hand, the same reduction curves were used for all spectral subdivisions. Although the line intensities vary considerably, the new reduction curves make the ordinary giants of all spectral subdivisions of nearly the same visual absolute magnitude. The small dispersion in absolute magnitude for these stars is quite remarkable. This cannot be due to a failure of the spectral criteria, for, as seen by the diagram, the variation of $\overline{M}$ with respect to $\overline{M}_1$ is practically linear throughout the entire range of absolute magnitude. If we compute the dispersion in absolute magnitude among the supergiants and ordinary giants separately by assuming the absolute magnitude to be constant, we find by formula (10), (where τ' now is equal to $\tau\, 10^{0.2m}$), $q = 1.17$ and 0.70 mag., respectively, corresponding to the probable errors of ± 0.79 and ± 0.47 mag. This means that the dispersion in the ordinary giants is so small that, if we could separate them from the supergiants, which have a much larger dispersion, we should attain nearly the same accuracy by giving them all the same absolute magnitude.

Part of the error in the absolute magnitude arises from error in the measurement of the line-intensities. If we estimate the probable error from this source to be ± 0.2, a somewhat larger amount must be due to a lack of correspondence between the spectral criteria and the absolute magnitudes.

The most important question, however, concerns the reliability of the systematic corrections. To determine the corrections, parallactic motions have also been used, but the results are very uncertain, mainly because of the large dispersion in velocity. The solar motion as computed from a small number of radial velocities has little weight, and, similarly, the group motion for a small number of objects may differ greatly from that derived from very large groups. Further, the underlying assumption is quite different from that used in dealing with peculiar motions. The assumption usually made is that the linear group motion is known and that the sum of the squares of the peculiar angular motions is a minimum. When the dispersion in linear motion is large, the result must be very uncertain, and may even be systematically affected. Systematic errors in the proper motions are also very serious, since the motions themselves are very small. Further, in using peculiar motions, all stars

have the same weight, which is not at all the case when the parallactic motions are employed.

In the present case, we find that the absolute magnitudes computed from parallactic motion, with a solar motion of 23 km/sec., would be brighter by 1.02 for the supergiants and 0.49 for the ordinary giants. A similar discrepancy was found for the long-period variables.[1]

The adopted value of the average error in the proper-motion components, which was supposed to be $\pm 0''.003$, may be too small. If we assume an error of $\pm 0''.008$, the effect on the magnitudes of the two classes of stars would be to make them 0.12 and 0.05 mag. brighter, respectively, than the published values. The uncertainty from this cause is therefore not serious.

The trigonometric parallaxes are very small and not numerous enough to determine the mean absolute magnitude with the same accuracy as can be derived from the peculiar motions. Using the parallaxes compiled by Schlesinger, we find the systematic correction to the provisional system to be +0.24 mag., which is in good agreement with the results from peculiar motion. For an assumed probable error of $\pm 0''.007$ in the trigonometric parallaxes, the corresponding probable error in the absolute magnitudes is ± 0.3. A larger probable error in the trigonometric parallaxes would make the probable error in the absolute magnitudes even smaller. But all results from the trigonometric parallaxes are rather uncertain, and it seems better to reverse the process and determine the errors in the trigonometric parallaxes from the spectroscopic values.

Although the cause of the systematic difference between the results based upon peculiar and parallactic motion is still obscure, there can be little doubt about the small dispersion in the visual absolute magnitudes. Moreover, a systematic correction as large as 0.5 mag. cannot be applied to the absolute magnitudes without producing a serious discrepancy between the dispersion in tangential and radial velocities, which is of special importance when the parallaxes are used for computing space velocities.

MOUNT WILSON OBSERVATORY
December 1926

[1] *Mt. Wilson Contr.*, No. 267; *Astrophysical Journal*, 59, 97, 1924.

NOTE ON THE PHYSICAL THEORY OF METEORS

By F. A. LINDEMANN

ABSTRACT

Criticisms advanced by Dr. Sparrow against a theory of meteors proposed by the writer and Dr. Dobson in 1922 are dealt with. The main point of divergence between the two theories lies in considering the mechanism of the heating of the meteor. Direct impacts of the air molecules upon the surface of the meteor are supposed by Dr. Sparrow, whereas the shielding of the meteor by the cap of gas is considered by the writer and Dr. Dobson. The latter consideration forces the conclusion that the temperature of the highest regions of the earth's atmosphere is higher than that of the stratosphere.

Dr. Sparrow's theory leads to impossibly large dimensions of meteors. Also the data obtained by means of *ballons sondes* agree closely with the calculations by the present writer. The densities of the air at the same height derived from the height of appearance and disappearance of the meteor are in good agreement, a fact which definitely rules out Dr. Sparrow's fundamental assumption of the direct molecular impacts.

General physical considerations such as the process of dissociation of the air and meteoric molecules, reabsorption of the radiated energy by the meteor, etc., are not consistent with Dr. Sparrow's theory. His objection against application of the idea of adiabatic compression to the case of meteors and criticism of numerical calculations from this theory are not convincing.

In the issue of the *Astrophysical Journal* for March, 1926, Professor Sparrow published an article entitled "The Physical Theory of Meteors." Since in this paper he criticizes somewhat adversely various parts of the theory published by the writer and Dr. Dobson in the *Proceedings of the Royal Society* in 1922, it is probably desirable to reply. This is the more easy as his theory is the most elementary and obvious one that could be constructed, and one which we had considered in the first instance and had been obliged to reject. The reason for this Professor Sparrow would probably have discovered for himself had he read our paper earlier and realized that his theory had already been tried and found wanting. Unfortunately, it appears "that the details of his theory had been fully worked out before he was able to obtain access to our paper."

Briefly the two theories may be summarized as follows. Dr. Sparrow believes, as we do, that a meteor appears when the surface of a particle entering the air at high speed is sufficiently heated to produce copious evaporation. The main point of difference is that Dr. Sparrow believes this heating to take place by the direct impacts of the molecules in the upper atmosphere upon the meteor, whereas we, on the other hand, suggest that heating is inappreciable until a

cap of gas forms in front of the meteor and that the heat is transmitted only comparatively inefficiently through this cap of gas on to the surface. The different results to which the two theories lead is caused by the fact that according to Dr. Sparrow's view a large fraction of the total energy is imparted to the meteor and becomes available for evaporation, whereas in our opinion, owing to the shielding of the cap of gas, a comparatively small fraction only of the available energy actually reaches the surface of the meteor and gives rise to evaporation. Thus Dr. Sparrow can make shift with a comparatively small number of molecules, i.e., a low density of the upper air, whereas we are obliged to conclude that the number of molecules is very much greater—in other words, that the density of the upper air is very much higher than that assumed by Dr. Sparrow. Since such high density can be obtained only if the temperature is comparatively high, we are therefore forced to conclude that the temperature in the uppermost regions of the earth's atmosphere is considerably higher than it is in the stratosphere.

There are two lines upon which one can approach such a problem. The first is to consider which theory is better supported from the point of view of meteoric phenomena. The second is to examine which theory is more likely to be true on general physical grounds.

If we consider the general phenomena associated with meteors it may readily be shown that Dr. Sparrow's theory can be ruled out. The contradiction is perhaps most easily seen if one examines the consequences to which it leads in the case of an average meteor, e.g., one traversing a path of 60 km in 1.5 seconds and appearing at a distance of 150 km as bright as a first-magnitude star. It may be shown by the most elementary methods, since it is agreed that the light is caused by the deceleration of evaporated meteoric matter, that in this case the rate of evaporation must be of the order of 4 mg per second. Now according to Dr. Sparrow's view, in spite of this comparatively rapid rate of evaporation in front of the meteor, the surface has still to be subjected to direct molecular impacts on the part of the gases composing the upper air. If this is to be so it is clear that the surface of the meteor must be very large, for unless the 4 mg a second which are produced by the meteor are spread over a very large volume, no molecule from the outer air will be

able to penetrate to the surface of the meteor without collision. A rough estimate shows that the meteor would have to have a radius of the order of 5 or 10 cm., i.e., a mass of the order of 1 kg. It is clear that such assumption is absurd, for since it is not appreciably decelerated and is only evaporating at a rate of 4 mg a second, such a meteor could not disappear after 1.5 second, but must inevitably reach the ground.

Our method of calculation may be checked in a very simple manner. At low heights the temperature of the air has been determined with *ballons sondes* and the density therefore is known exactly. On the other hand, some meteors have been observed at these heights and have been used to calculate the density by means of our formula. The density of the air determined by our calculations coincides closely with the directly observed densities, so that there seems no reasonable doubt that the general line of attack is correct.

There might still be a doubt whether the preliminary heating before the meteor begins to evaporate takes place by direct impact as suggested by Dr. Sparrow, or whether it becomes appreciable only after the formation of the cap of gas, as we have assumed. Here, fortunately, a simple check is available. If Dr. Sparrow were right, heating should be highly efficient in the early stages where direct impacts obviously occur; hence the densities of the air calculated by our formulae from the heights of appearance should be very much less than the density of the air calculated for the same height from the heights of disappearance, since we have neglected the heating in the stage of direct impact. A glance at the figure in our papers shows that the densities calculated from the heights of appearance agree closely with the densities calculated from the heights of disappearance of the meteor. As has been shown above, there can be no reasonable doubt that the density calculated from the heights of disappearance are correct, and therefore we must assume that the method of calculating the densities from the heights of appearance which results in complete agreement must also be right. It follows that we cannot accept Dr. Sparrow's view that any appreciable heating of the meteor takes place by direct impact even in the early stages before evaporation has commenced.

This reason, apart from any others, compelled us definitely to

abandon any theory of the nature of that now put forward by Dr. Sparrow. There are, of course, numerous other difficulties which would arise if this theory were correct. For instance, according to his theory all meteors of equal velocity appear at the same height but should disappear at different heights, whereas there seems to be a definite correlation between the heights of appearance and disappearance caused, in our view, by the fact that a large meteor takes longer to heat up than a small meteor, and therefore appears lower down and also disappears lower down. It is not, however, in view of the fundamental and conclusive objection to Dr. Sparrow's theory outlined above, necessary to go into these matters.

It may, nevertheless, be interesting to revert to the general physical grounds which led us to believe that only a small fraction of the energy produced by direct molecular impact would be transferred to the meteor. Dr. Sparrow admits that a large fraction will go in energy of dissociation of the atmospheric molecules, as also that the energy of atomic ionization will not be available for heating the meteor. On the other hand, he claims that the energy of dissociation of the meteoric molecule or of the disruption of its constituent parts should go entirely toward heating the meteor. Frankly, this seems difficult to understand, for surely this energy is used up precisely in dissociating meteoric molecules and not in heating the meteor. One cannot heat ice by placing it in a high vacuum and causing it to evaporate, nor can one heat a meteor by causing the meteoric molecules to evaporate or dissociate.

Dr. Sparrow then claims that of the energy radiated at the surface of the meteor approximately half is reabsorbed by the meteor. We will not go into the question of the reflection coefficient, but surely most of the heat is reabsorbed in the immediate neighborhood of the spot where the molecular impact took place and will be reemitted, and this process will continue until the rate of emission is of the same order as the rate of heat conduction, i.e., until comparatively low temperatures, or, in other words, until all the energy has been dissipated.

Finally, Dr. Sparrow objects that we have taken the velocity of elastic waves as the upper limit for the speed with which energy is transferred. The amount transferred by radiation would be almost

negligible in a substance so opaque as a meteor. The velocity of transference by free electrons, which Dr. Sparrow remarks is comparable to the rapidity of metallic conduction, is, it would seem, completely unknown. Unless he can throw some much-needed light upon this difficult subject he can scarcely expect us to accept his criticism.

There is one other point to which Dr. Sparrow seems to attach importance since he mentions it in his abstract, and that is that he believes it to be wrong to take the temperature of adiabatic compression as the temperature reached by the gas compressed in front of a meteor. His arguments on this subject appear quite unconvincing. He seems to forget that the relative speed between the surface and the gas cannot exceed the velocity of the molecules, once the gas is sufficiently dense for the free path of the molecules to be small compared to the linear dimensions of the surface. If it did, all the molecules would be compressed against the surface and we should have no gas.

What really happens is that a cap is formed in front of the surface in which the relative speed diminishes by something of the order of the molecular velocity for each free path, so that the outer layers of the cap are moving comparatively slowly relative to the gas through which the particle is passing, whereas the inner layers are moving comparatively slowly compared to the surface of the particle itself. In these circumstances the ordinary laws of adiabatic compression will not be far wrong. The temperature at any one point is defined by the relative molecular velocity, not by the velocity relative either to the air or to the particle, and consequently the temperature is nothing like as high as believed by Dr. Sparrow.

The last criticism of Dr. Sparrow to which it may be worth while to revert is the one in which he claims that our figures are inconsistent in that the temperature which we said was of the order of 300° leads to an increase of density by a factor of only about 100 instead of the order of 1,000. If Dr. Sparrow looks at the figures once more and has due regard to the spreading of the points and to the uncertainty of the data on which they are based, he will realize that we should not venture to give any figures so exact as he imagines. All we were concerned to show was that the density was very much

higher than was generally assumed, and to indicate that this led, in our view, to a temperature considerably higher than that which had generally been expected.

Dr. Sparrow appears to think that there is some special assumption involved in saying that the temperature is of the order of 300°, whereas there is no assumption involved in saying that it is about 220°. This is a point upon which it seems difficult to compromise. Because the temperature between 10 and 25 km has been observed to be 220°, this is no reason whatsoever for assuming that the temperature at 100 km is 220°. There is just as big an assumption in saying that the temperature remains constant as there is in saying that the temperature rises, and when, as has been shown, a considerable amount of heat is absorbed by the ozone in the upper air, whereas practically none is absorbed in the lower regions, it seems that the suggestion that the temperature rises has some plausibility; whereas the hypothesis that the temperature remains constant has nothing to recommend it beyond the fact that it saves trouble in computation.

It would seem that this point has scarcely been sufficiently realized. If it had, Dr. Sparrow could not have claimed that the advantage of his theory is that it requires no extensive modification of our current notions.[1]

While it may be admitted that some of these physical points are doubtful, the fatal and final objection to Dr. Sparrow's theory seems to remain beyond argument. If his theory were applicable down to low heights, then the densities calculated at 20 and 30 km at which immediate observation is possible would be completely wrong. If, on the other hand, his theory applies, as it obviously can apply, only to the early stages before evaporation has set in, then the densities calculated from the heights of appearance and disappearance could not coincide. In our view we have a similar mechanism heating the meteor throughout. Whether one accepts or denies our derivation, the efficiency factor in heating the meteor may be taken directly from the known densities of the air at low heights. If one takes this same efficiency factor and applies it throughout the

[1] Incidentally this agreement is attained only by assuming an improbably high value for the concentration of H_2 in the atmosphere.

range, one obtains the densities we have found from the heights of disappearance of the meteor and these coincide with the densities calculated from the heights of appearance of the meteor.

Any new theory which endeavors to supersede ours must take these facts into account; above all, it must avoid inherent contradictions such as the theory of Dr. Sparrow involves, and which led us to abandon his hypothesis in the very early stages of our consideration of the subject.

CLARENDON LABORATORY, OXFORD
November 27, 1926

THE SHORT-PERIOD VARIABLE STAR RV CANUM VENATICORUM[1]

By J. SCHILT

ABSTRACT

Light-curve.—From ninety-one plates the light-curve of RV Canum Venaticorum is found to be of the type of W Ursae Majoris, with a period of 0.269572 day. This makes it highly probable that the star is a dwarf eclipsing binary, instead of a variable of the cluster type, as stated by Graff.

Spectrum and range in light.—The spectrum is F8; the photographic range is 0.75 mag. and apparently does not exceed the visual range.

The variability of this star, which is 25′ from the center of the globular cluster M_3, was first detected by J. Larink,[2] who found a period of 0.134786 day. A photometric visual light-curve has been published by K. Graff,[3] who calls attention to many irregularities. The reality of these irregularities seems doubtful. I have combined Graff's observations into groups of three. The dispersion of the deviations of such normal points from a symmetrical smooth curve is $\pm 0.062 \pm 0.019$ mag. (mean error). Graff gives a mean error of ± 0.06 mag. for his individual observations, which, if the latter are independent, amounts to ± 0.036 mag. for the normal points. The difference between the observed dispersion and the value given is 0.026 ± 0.019 mag. Whether this is accidental or a consequence of some systematic error in Graff's observations cannot be settled. It will be seen, however, that the present observations do not corroborate the alleged irregularities.

To determine the photographic light-curve, ninety-six plates, each with eight minutes' exposure, were taken, mainly during four nights, with the 60-inch reflecting telescope. The plates used are Eastman 40, size 4 by 5 inches, so that the center of the globular cluster M_3 is just on the following edge of the plates. Ninety-one of these plates have been measured with the thermopile microphotometer. The magnitudes of the comparison stars were derived

[1] *Contributions from the Mount Wilson Observatory*, No. 330.

[2] *Astronomische Nachrichten*, **214**, 71, 1921.

[3] *Ibid.*, **217**, 310, 1923.

from the comparison plates SS 145, 146, and 147, which have the following exposures and zenith distances:

Plate	First Exposure	Second Exposure
SS 145	Selected Area 57, 15°	RV Can. Ven., 12°
146	RV Can. Ven., 13	Selected Area 57, 8
147	Selected Area 57, 5	RV Can. Ven. 4

Because of the small zenith distances, no correction for differential extinction is required. The magnitudes, on the international scale, of the stars of the Selected Area were kindly supplied by Mr. Seares, and the resulting magnitudes of the comparison stars, together with their rectangular co-ordinates, are entered in Table I.

TABLE I

VARIABLE AND COMPARISON STARS

Star	*X*	*Y*	Pg. Mag.	
			Provis.	Adopted
RV Can. Ven.	0″	0″	.....	.....
a	+358	+35	15.74	15.78
b	320	41	15.33	15.38
c	+274	30	14.72	14.59
d	−279	37	16.20	16.16
e	−417	+ 8	14.62	14.71

The provisional values were derived directly from the three comparison plates, whereas the adopted magnitudes are the smoothed values found by using measurements from all the plates. The comparison stars *c* and *e* happen to be the same as Graff's stars *f* (vis. mag., 13.41) and *a* (vis. mag., 13.66), respectively.

The observational data are entered in Table II. The observations have been combined into groups of four in order of phase. The normal points thus obtained are given in Table III, and are shown graphically in Figure 1. The computed minimum is at phase 0.185 *P*. The mean magnitude at maximum from the eighteen observations between phases $0.685 \pm 0.100P$ is 14.897 ± 0.014 mag. (mean error); from the eleven observations between the narrower limits 0.685 ± 0.050 it is 14.883 ± 0.017 mag. (mean error). The adopted maximum light is 14.88 mag. The magnitude at minimum is estimated to be 15.63 ± 0.04 mag., from which the photographic range is 0.75 ± 0.04 mag.

TABLE II

The Observations

Plate	J.D. Hel. G.M.T. 2424000+	Epoch and Phase	Mag.	Plate	J.D. Hel. G.M.T. 2424000+	Epoch and Phase	Mag.
SS 64..	564.9255	0.000*P*	15.33	SS 128..	620.9386	415.571*P*	14.85
65..	.9330	.055	15.38	129..	.9455	.622	14.87
66..	.9400	.106	15.56	130..	.9524	.673	14.83
67..	.9469	.158	15.69	131..	.9593	.724	14.90
68..	.9538	.209	15.70	132..	.9766	.853	14.94
69..	.9608	.261	15.61	138..	621.7003	421.222	15.55
70..	.9677	.312	15.38	139..	.7072	.273	15.44
72..	.9816	.415	15.23	140..	.7142	.325	15.10
73..	.9885	.467	15.06	141..	.7211	.376	15.02
74..	.9954	.518	15.11	142..	.7280	.427	14.94
75..	565.0024	.569	15.15	143..	.7349	.470	14.82
76..	.0093	.621	14.89	144..	.7418	.530	14.90
77..	.0163	.672	14.95	145..	.7883	.875	14.86
78..	.0232	.723	14.93	146..	.7953	.927	15.06
79..	.0301	.775	14.95	147..	.8243	422.142	15.60
80..	.0371	.826	14.78*	149..	.8804	.558	14.93
81..	.0440	.878	15.03	150..	.8874	.610	15.04
82..	.0510	.930	15.02	151..	.8943	.661	14.98
83..	.0579	0.981	15.06	152..	.9012	.712	14.87*
98..	619.9025	407.884	15.02	153..	.9082	.764	14.91
99..	.9140	.969	14.90	154..	.9151	.816	14.94
102..	620.7307	414.029	15.25	155..	.9220	.867	14.97
103..	.7377	.080	15.34	156..	.9289	.918	15.10
104..	.7446	.131	15.58	157..	.9358	.969	15.13
105..	.7515	.182	15.67	158..	.9428	423.021	15.18
106..	.7585	.234	15.58	159..	.9497	.072	15.20
107..	.7654	.286	15.36	171..	642.6838	576.902	14.86
108..	.7724	.338	15.24	172..	.6908	.954	15.07
109..	.7793	.389	15.13	173..	.6977	577.005	15.19
110..	.7862	.440	15.03	174..	.7046	.056	15.19
111..	.7932	.492	14.94	175..	.7115	.108	15.31
112..	.8001	.543	14.89	176..	.7184	.159	15.39
113..	.8070	.594	14.86	177..	.7253	.210	15.55
114..	.8140	.646	14.82	178..	.7323	.262	15.28
115..	.8209	.697	14.86	179..	.7392	.313	15.18
116..	.8278	.749	14.83	180..	.7461	.364	15.10
117..	.8347	.800	14.84	181..	.7530	.415	15.00
118..	.8416	.851	14.95	182..	.7600	.467	14.98
119..	.8486	.903	14.88	183..	.7669	.518	14.95
120..	.8555	.954	15.10	184..	.7738	.570	14.94
122..	.8971	415.263	15.28	185..	.7807	.621	14.91
123..	.9040	.314	14.95	186..	.7877	.673	14.84
124..	.9110	.366	15.11	187..	.7946	.724	14.90
125..	.9179	.417	15.04	188..	.8113	.848	14.89
126..	.9248	.468	14.97	190..	642.8535	578.161	15.50
127..	.9317	.520	15.01				

* Taken through the glass.

The observations by Graff have been plotted as crosses in Figure 1, each cross representing the mean of three successive observations. In order to make the ordinates correspond to the photographic light-curve, a constant, +1.35 mag., has been applied to Graff's visual observations. The magnitudes at maximum, 13.48±0.03 mag., and at minimum, 14.29±0.05 mag., give a visual range on

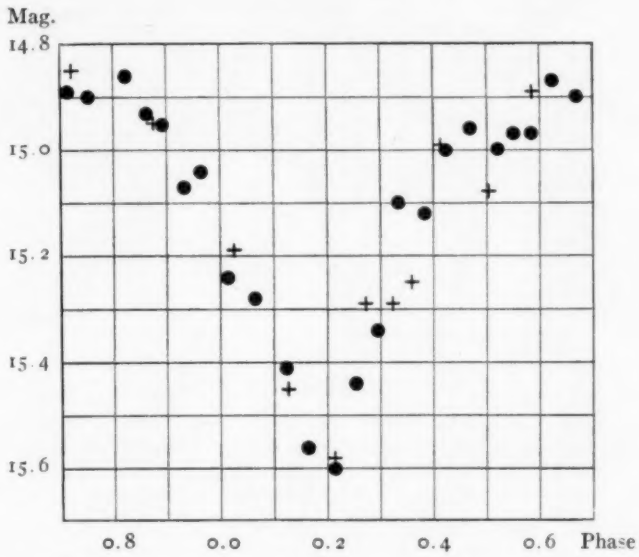


FIG. 1.—Observations of RV Canum Venaticorum in groups of four (dots) plotted according to phase as calculated from Larink's period (half-period). Crosses represent means of three consecutive observations by Graff, reduced to the zero point of the Mount Wilson measures.

Graff's scale of 0.81±0.06 mag. The difference between the photographic and visual amplitudes is thus −0.06±0.075 mag., from which we may conclude that, on the basis of the adopted scales, the photographic amplitude does not exceed the visual range.

It is evident from the measures that the light-curve is of the type of W Ursae Majoris, and this conclusion is not contradicted by Graff's observations.

The spectrum obtained with a slitless spectrograph at the 60-inch reflector is F8. The lines seem to be diffuse, though there is some uncertainty on account of the low dispersion used.

The data so far obtained indicate that RV Canum Venaticorum is a very faint representative of the W Ursae Majoris class of stars, and presumably a dwarf. As it is an eclipsing binary, Larink's period

TABLE III

NORMAL POINTS

Phase	Mag.	Phase	Mag.
0.014*P*	15.24	0.550*P*	14.97
.066	15.28	.586	14.97
.122	15.41	.628	14.87
.165	15.56	.670	14.90
.212	15.60*	.714	14.89
.255	15.44	.753	14.90
.296	15.34	.822	14.86
.335	15.10	.862	14.93
.386	15.12	.892	14.95
.425	15.00	.932	15.07
.470	14.96	0.968	15.04
0.512	15.00		

* Mean of three observations.

TABLE IV

NORMAL POINTS FOR REVOLUTION PERIOD

Phase	Mag.	Phase	Mag.
0.006*P*	15.61	0.514*P*	15.40
.057	15.40	.553	15.21
.122	15.11	.586	15.08
.172	14.97	.624	14.99
.206	14.98	.656	14.94
.242	14.90	.694	14.90
.276	14.88	.744	14.86
.310	14.88	.820	14.90
.344	14.95	.871	14.99*
.370	15.02	.928	15.32
.396	15.11	0.974	15.61
0.440	15.22		

* Mean of three observations.

must be doubled. New normal points for the complete period of revolution are entered in Table IV, where the phase is now counted from the primary minimum.[1] The light-curve for the double period,

[1] It is suspected that the comparison star *e* may be variable. The magnitudes were therefore determined a second time without using star *e*. The average difference, without regard to sign, between the two sets of values is 0.02 mag. The results of the first reduction have been retained.

shown in Figure 2, has slightly different minima and an asymmetry which, even if real, need cause no surprise, since it is quite comparable with asymmetries found in other stars of this class.

The heliocentric time of principal minimum derived from the present observations is J.D. (Greenwich Mean Time) 2424642.5863,

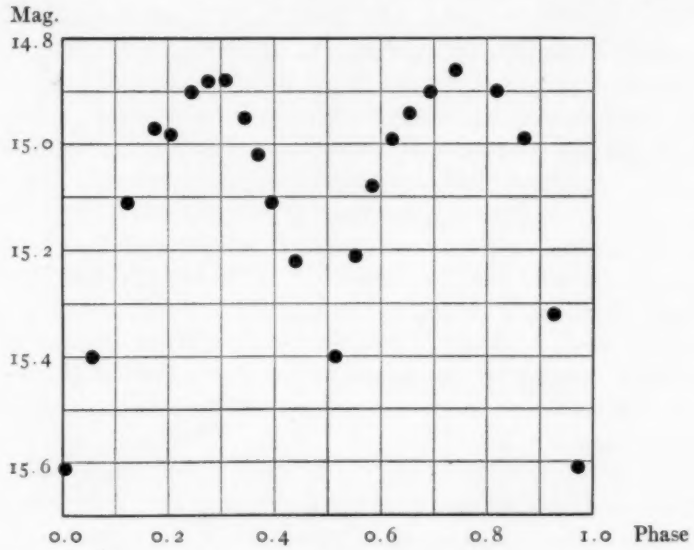


FIG. 2.—Normal points for RV Canum Venaticorum corresponding to complete period of revolution.

while that of the minimum observed by Graff is 2422811.5540. Since the period is known only with an accuracy of one second,[1] the interval between the two epochs is, unfortunately, too large to determine the number of intervening revolutions. The revised elements are:

Min. (J.D. Hel. G.M.T.) = $2424642.5863 \pm 0.269572E$
$\pm .0020 \pm 0.000023$ (m.e.)
Maximum light . . . . . 14.88 ± 0.02 mag. (m.e.)
Primary minimum . . . . . 15.65 ± 0.10
Secondary minimum . . . . 15.48 ± 0.10

where the period is Larink's value doubled.

MOUNT WILSON OBSERVATORY
August 1926

[1] J. Larink, *loc. cit.*

REVIEWS

Elements of Astronomy. By EDWARD A. FATH. New York: McGraw-Hill Book Co., 1926. 8vo. Pp. viii+307. Figs. 191. $3.00.

In connection with the recent progress that has been made in astronomy, teachers of the subject have felt a growing need for a textbook that would give an adequate presentation of this new material. Professor Fath has admirably met such a need in this publication. This book is intended for use in elementary courses and presents, in a very clear and concise manner, not only the main facts and theories of astronomy formerly accepted, but includes a description of the most noteworthy contributions made during the recent years by such investigators as Shapley, Kapteyn, Russell, Eddington, Seares, Hubble, and others.

The book consists of nineteen chapters, covering all portions of the subject suitable for study by college students without mathematical prerequisites.

While the descriptions and explanations are necessarily brief in order to cover such a large field in so few pages, they give the essential details in such a manner as to be easily understood by the general reader. Such topics as "The Nature of Light" including the Bohr theory of the atom, "The Precession of the Equinoxes," and "The Michelson-Gale Experiment" are examples of the skill with which the author handles difficult topics. Attention should be called to the error in the first edition of the statement of the result of the Michelson-Gale experiment. The final result, published in the *Astrophysical Journal* for December, 1919, states that the rigidity of the earth in the north-south and east-west directions is the same.

The presswork is excellent, and the illustrations consisting of halftones and line drawings are well selected and show to good advantage on the smooth-finish paper, which is of good quality and about right in weight. One wonders, however, why the illustration of the types of stellar spectra published in Volume III of the *Detroit Observatory Publications* is not used in place of Figure 143. Certainly Figure 62, showing the moon near full, should be placed on the page opposite Figure 63, showing the outline map of the moon. It would also help the student if Figure 48 showed the path of the violet and red rays, if both branches of the

hyperbola were shown in Figure 76, and if the angles Ω and ω were indicated in Figure 114. In the text accompanying this figure, an error is made in using γ in place of Υ.

It is to be hoped that in a later edition the author may add some simple exercises, a bibliography, and an appendix containing reference tables of historical, planetary, and stellar data. Also it would increase the value and interest of the book if more historical matter were given, such as Eratosthenes' method of determining the size of the earth, a diagram of the Ptolemaic system, etc. In the chapter on "The Earth," the only proof of the earth's revolution around the sun given is that based on the aberration of light. Two or three other proofs might be given here also. The general reader will hardly get an adequate idea of the nature of sun-spots when he reads on page 107, "Spots, in general, are vortices with a downward suction."

The foregoing criticisms are of minor importance, however. On the whole, the book is very well done and the author has achieved a distinct success in providing a non-mathematical presentation of the main facts of astronomy and a simple and interesting description of the principles and methods used in modern astronomical investigation.

W. H. GARRETT

Die Kultur der Gegenwart, Dritter Teil, Dritte Abteilung: PHYSIK. Zweite Neubearbeitete und Erweiterte Auflage. Edited by E. LECHER. Leipzig and Berlin: B. G. Teubner; 1925. 4to. Pp. 849. Bound, M. 36.

The first edition of this work, edited by Professor E. Warburg, appeared in 1915, the Preface having been dated July, 1914. This new edition has been revised and extended to include the notable progress in Physics in the following decade distinguished by remarkable progress. The editor of the new edition, Professor E. Lecher, has quite closely followed the arrangement of the subject matter adopted by Professor Warburg.

The first edition was reviewed by Professor Henry G. Gale in the issue of this *Journal* for April, 1916 (**43**, 250), where a list of the separate authors and the subjects of their chapters was given. The new edition contains a further chapter of 16 pages, on "Quantentheorie, Atombau und Spektrallinien," by H. A. Kramers. Five authors of sections in the first edition have passed away in the interval between the two editions, and

their contributions have been revised by other experts as follows: H. Rubens, "Wärmestrahlung," by G. Hettner; F. Braun, "Die Drahtlose Telegraphie," by M. Diekmann; J. Elster and H. Geitel, "Entdeckungsgeschichte und Grundtatsachen der Radioaktivität," by St. Meyer and E. Schweidler; O. Lummer, "Neuere Fortschritte der geometrischen Optik," by M. v. Rohr; F. Hasenöhrl, "Die Erhaltung der Energie und die Vermehrung der Entropie," by H. Mache.

The revision has increased the number of pages of the book from 762 to 849, and ten new illustrations have been added, making a total of 116. It will be recalled that this work was intended for the academic reader and for specialists in related fields of science. Therefore, few technical formulae are included, and the treatment is general and philosophic.

To those interested in astrophysics, we can especially recommend the new chapter by Kramers, as well as the following: "Experimentelle Atomistik," by Dorn and Przibram; "Theoretische Atomistik," by Einstein; "Wärmstrahlung," by Rubens and Hettner; "Die positiven Strahlen oder Massenstrahlen," by Gehrcke and O. Reichenheim; "Die Röntgenstrahlen," by Kaufmann, the chapter originally by Elster and Geitel which we have already named; "Entwicklung der Wellenlehre des Lichtes," by Wiener; "Spektralanalyse," by Exner; "Struktur der Spektrallinien," by Gehrcke; "Magneto-optik," by Zeemann.

The book is handsomely printed on good paper, and the marginal designation of the separate divisions of the chapters is very convenient.

F

Fundamental Concepts of Physics. By PAUL R. HEYL. Baltimore: Williams & Wilkins Co., 1926. Pp. xii+112. Figs. 3. $2.00.

This attractive little volume contains three lectures given at the Carnegie Institute of Technology, Pittsburgh, in January, 1926. The three lectures are entitled: "The Eighteenth Century: The Century of Materialism," "The Nineteenth Century: The Century of Correlation," "The Twentieth Century: The Century of Hope." It is well for those engaged in research as well as for those whose time has to be chiefly spent in teaching to pause and consider, even briefly, the history and philosophy of physics. The author has done his work well, with accuracy and with imagination. Some may think that the latter is carried too far in the third chapter, but the twentieth century is still young, and the greater part of its achievements lie before us. At the present stage, it may therefore well be called, "the century of hope."

F

Amateur Telescope Making. By RUSSELL W. PORTER, WILLIAM F. A. ELLISON, AND OTHERS. New York: Scientific American Publishing Co., 1926. Pp. x+102. Illustrated. $2.00.

This is an admirable booklet for amateurs on the making of mirrors and mountings for reflectors, with a minimum of theory and a maximum of practical directions based on experience. It includes articles by Porter reprinted from the *Scientific American*, portions of Ellison's *The Amateur's Telescope*, and a chapter on silvering. Other chapters tell how to make a mounting from discarded automobile parts, how to make a simplified reflector, where the amateur may get his materials, and how much they will cost. The directions are simple and, within the scope of the book, complete. A bibliography, with comments, adds greatly to the value of the book.

S. B. B.

Tabelle der Hauptlinien der Linienspektra aller Elemente. By H. KAYSER. Berlin: Julius Springer, 1926. Pp. vi+198. M. 24.

Observational work in spectroscopy in the past has undoubtedly been accelerated by easy access to information about the work of others. By far the greatest amount of such information has been made available through the work of Kayser, to whose monumental compilations has been added yet another.

As the author points out in his Preface, the table of "Hauptlinien" at the end of Volume 6 of the *Handbuch der Spectroscopie* has become out of date. In the decade and a half since its publication, more precise measurements in the region of the visible spectrum, and extensions to longer and shorter wave-lengths have been made until the region now covers from about 100 A to 9 μ. Further, the increase in knowledge of spectral series relations has led to a new sort of classification according to successive stages of excitation, so that instead of a single spectrum for an element we now distinguish between a number of spectra. For instance, following commonly accepted usage, a line ascribed to the spectrum of the neutral atom of silicon is indicated by *Si*I; one belonging to the spectrum of the singly ionized atom, by *Si*II, and so on. Kayser estimates that 15 per cent of the lines listed in this new table of the principal lines are correctly classified in this manner.

The table contains 19,000 wave-lengths in the International system, with the intensity and origin in arc, spark, and vacuum tube for each line. Special characteristics of lines are included where they have been ob-

served. The table begins at 9.0850 μ and ends at 124.0A. For wave-lengths longer than 1 μ the name of the observer is given; for wave-lengths shorter than 2000A, this has also been done for lines measured by only one observer. As explained in the Preface, the choice of a value for the wave-length when observers differ is solely a matter of the character of the observations and the good judgment of the author. The scale of intensities is not intended to be anything but a rough indication of the strength of the lines. It is unfortunate that in many instances the classification of lines as appearing in arc, spark, or vacuum tube is inadequate, but nothing more can be expected in view of the widespread origin of the available information. A table is included by which wave-lengths measured in the Rowland scale can be corrected to the International system, covering a range from 7000 to 1950A. The author points out that extrapolation in either direction is unsafe; indeed, comparatively few observations outside those limits were made before the common acceptance of the International system. In the table of symbols of the elements are some welcome improvements, such as the use of "*Em*" for Emanation and "*Nt*" for Neothulium.

With a realization that this book, in turn, will become out of date, the tables have been printed on good writing paper, and ample margins left for the entry of additional data. The volume is an invaluable addition to the facilities of the observational spectroscopist.

GEORGE S. MONK

Practical Physics. By T. G. BEDFORD. London: Longmans, Green & Co., 1926. Demi 8vo. Pp. x+425. $3.50.

This book is an admirable compilation of more than one hundred and fifty experiments in all branches of physics as provided for the first-year students at the Cavendish Laboratory, Cambridge. The theory as well as the details of the manipulations are discussed in every case, and for some experiments several alternative methods are presented. An introductory chapter contains advice to the student on laboratory practice, units, errors of observation, and the representation of results by means of graphs. All of the experiments usually performed in junior college courses are discussed, and in addition "a few less easy experiments have been included, to afford scope to students who are in a position to omit some of the elementary work." The inclusion of several tables of the more common physical constants would make the volume a little more useful, even though standard tables are available in the laboratory.

Practical Physics sets a high standard for elementary laboratory work in physics. It should be in the hands of the honors students in every laboratory, and should prove of value to all teachers of general college-physics courses. It is probable, however, that our average American undergraduate would be confused and bewildered by the completeness and thoroughness of the detailed treatment.

WILLIAM W. WATSON

X-Rays and Electrons. By ARTHUR H. COMPTON. New York: D. Van Nostrand Co., 1926. Pp. xv+403. Figs. 132. $6.00.

According to its title-page this book is essentially "an outline of recent X-ray theory." It describes X-ray phenomena particularly from the point of view of their bearing upon the fundamental conceptions and laws relating to the structure of matter and the mechanism of radiation. The book, together with the new edition of Bragg's *X-Rays and Crystal Structure*, Wyckoff's *The Structure of Crystals*, the translation of Seigbahn's *The Spectroscopy of X-Rays*, Clark's *Applied X-Rays*, etc., covers very completely the field of X-radiation for the reader of English.

After a brief Introduction, describing the properties of X-rays, the first part of the book presents the explanations given by the laws of electrodynamics of the production of X-rays, the scattering of X-rays, the absorption of X-rays, the refraction of X-rays, and the reflection of X-rays, together with their use in determining the structure of crystals and the distribution of electrons in crystals and in atoms.

The second part of the book emphasizes the inadequacy of the classical theories and treats the subject from the point of view of the quantum theory. It describes the X-ray photo-electric effect; presents the quantum theory of X-ray scattering, of X-ray defraction, and of X-ray production and absorption; and gives a brief account of X-ray spectra and their bearing on Bohr's theory, etc. The book concludes with a number of appendices containing more highly mathematical treatments of some of the points mentioned in the text itself.

The text has been written in a very clear and interesting manner, with a reasonable use of elementary mathematics. Although it presents all of the important ideas and theories of X-radiation that have been published, it is of particular interest to the scientist because of its detailed descriptions of the author's own, very important, fundamental researches. The chief of these is, of course, the quantum theory of X-ray scattering, which

leads to the discovery of the change of wave-length due to the scattering process, known as the "Compton effect."

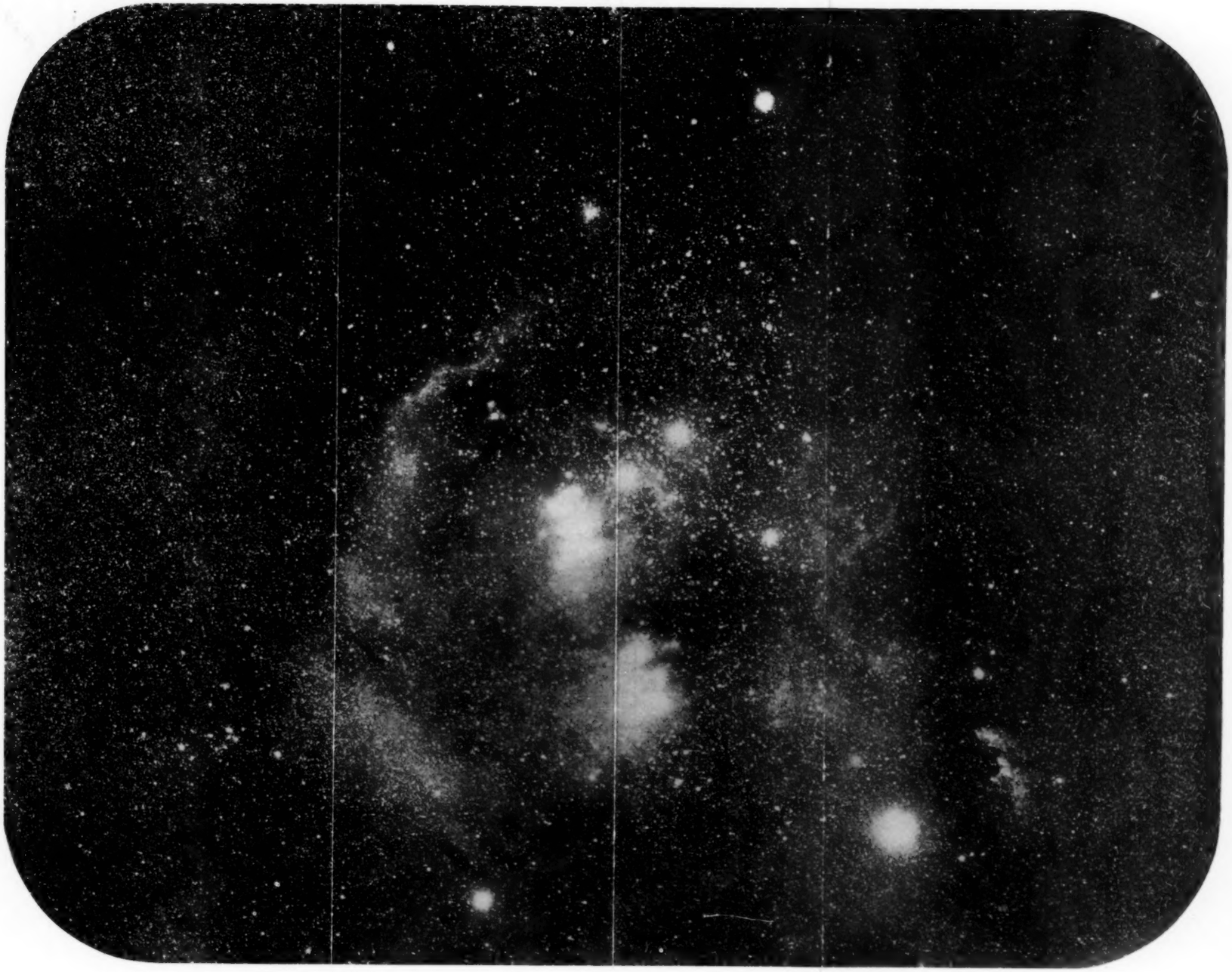
In describing researches other than his own, also, the author frequently presents the subject matter in a new and original manner. For instance, the Fourier series equations, which had previously been deduced from the quantum theory and used to calculate the distribution of electrons in crystals and atoms, are here developed from the classical theory.

The book has been well printed and contains a large number of well-drawn figures. The few errors in its text will doubtless be corrected in future editions.

No student of X-rays should fail to study this treatise.

WILLIAM DUANE

PLATE II



Orion and Its Nebulosities